

# Dealing with Overleverage: Restricting Leverage vs. Restricting Variable Compensation\*

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## Abstract

We study policies that regulate executive compensation in a model that jointly determines executives' effort, compensation and firm leverage. The market failure that justifies regulation is that executives are optimistic about asset prices in states of distress. We show that shareholders propose compensation packages that lead to socially excessive leverage. Say-on-pay regulation does not reduce the incentives for leverage. Regulating the structure of compensation (but not its level) with a cap on the ratio of variable-to-fixed pay delivers the right leverage. However, it is more efficient to directly regulate leverage because restricting the variable compensation impacts managerial effort more than if shareholders are free to design compensation subject to a leverage constraint.

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# 1 Introduction

Following the 2007-09 financial crisis, there has been a lively debate in the academic and policy circles about regulating executive compensation to avoid excessive firms' leverage. Some countries have regulated the structure or the level of compensation, especially for financial firms, while others have adopted say-on-pay regimes that increase shareholder's weight in the design of executive compensation. For example, the European Union (Directive 2013/36/EU and CRD4) has established that bonuses at credit institutions and investment firms cannot exceed 100% of fixed salary (200% if the company wins shareholder approval). The U.S. is also discussing new rules to curb executive compensation in financial institutions (Wall Street Journal 2016). Correa and Lel (2016) document that eleven countries have passed laws to give shareholders direct influence on executive compensation policies (i.e., say on pay laws).

In this paper we analyze policies that regulate executive compensation in a model that is new because it jointly determines leverage, compensation and executives' effort. This effort affects the likelihood of a crisis. The model yields three main insights. First, we show that, when the CEO is optimistic about asset prices in states of distress, shareholders prefer compensation contracts that induce socially inefficient firms' overleverage. Second, regulating the ratio of variable-to-fixed payments (but not the level of compensation) can deliver socially optimal leverage levels. However, our third result shows that it may be more efficient (i.e., less distorting in terms of effort provision) to directly regulate leverage rather than executive compensation.

In our model, a representative price-taker firm is run by a risk-neutral CEO ("she") who decides the firm's level of borrowing to finance an investment with stochastic payoffs. The model does not distinguish between financial and non-financial firms. There are many examples of overborrowing for both financial and non-financial firms. For example, Ryou and Kim (2003) describe overborrowing by Korean firms before the Asian financial crisis of the late 1990s. More recent examples include energy companies as Abengoa's debt-fueled expansion (Wall Street Journal 2015).

In the model, the CEO provides costly and unobservable effort that determines the likelihood of success of the investment. The firm's shareholder ("he") offers the CEO a compensation contract that includes, potentially, a fixed salary and a variable, performance-based bonus. Uncertainty is represented by two possible states. In the "low" (distressed) state of nature, the firm must sell core assets at a discount (i.e., fire sales) to cover debt losses. Following Gabaix (2014), the CEO overborrows because she underestimates the marginal cost of fire sales in the event of distress. This is what we define as managerial optimism.

The shareholder, even if he correctly estimates the marginal cost of fire sales, prefers not to amend the executive's optimism and tolerates overborrowing because of two reasons. First, higher leverage is motivating the CEO to put more effort making the good state of nature more likely; Second, the optimistic CEO, because she overestimates the firm's profits, ends up receiving a lower variable bonus than she expected. The shareholder benefits from an effort level higher than what he is ultimately paying for.

In the model, like in Krugman (1998), fire sales are not a mere wealth redistribution but imply real costs for society because in states of distress the assets end up being inefficiently

managed.<sup>1</sup> These "mismanagement externalities" caused by the fire sales creates a role for policy, even if the key friction is a behavioral one as optimism. The planner could achieve Pareto superior outcomes if it could reduce the market equilibrium level of fire sales. The planner would choose the socially optimal level of firm's borrowing as a tradeoff between the real costs of fire sales versus the gains from the investment financed with debt.

We analyze two policy tools to induce social efficiency. First, we restrict shareholders' choices on the structure (although not the level) of executive compensation. That is, we impose a cap on the variable relative to the fixed salary. Second, we directly regulate the leverage level, like with standard capital requirements or leverage restrictions. Finally, we compare the two policy tools.

The model shows that regulating the ratio of variable bonus to fixed salary may achieve the socially optimal level of debt. This policy tool reduces the CEO incentives to provide effort (induced by the variable bonus). Thus, the probability of a crisis may be higher, but the losses would be smaller due to smaller leverage.

Regulating compensation may not be the most efficient policy to tackle overleverage. It may be more efficient to directly restrict leverage as proposed, for example, by Korinek and Jeanne (2014). The intuition for this result is that restricting variable pay will likely distort effort incentives more than restricting leverage and letting the shareholder choose the compensation contract. Thus, variable pay will be higher under a leverage restriction and so will be managerial effort.

Our paper contributes to several literatures. First, a growing literature has documented that overconfidence and optimism by firms' executives leads to overinvestment and overborrowing (for example, Malmendier et al. 2005, Hackbarth 2009, Ben-David et al. 2013, Palmon and Venezia 2013 and 2015, or Ho et al. 2016). However, this literature has not studied the role of the endogenous CEO's effort. We show two non-trivial channels that make the CEO's effort increasing in optimism. First, there is a complementarity between effort and leverage. Optimism encourages higher leverage, and higher leverage entices higher effort to avoid the larger losses if the low state on nature is realized. This complementarity between debt and effort is new in the literature. Second, as the manager is compensated in equity, the manager has more incentives to put in effort at a more valuable firm. Thus, more optimism means more effort.

Second, we complement Gervais et al. (2011) who show that shareholders strategically benefit from managers who overestimate their own skills by "saving" on compensation.<sup>2</sup> In our case, executives overestimation is related to asset prices that we model as in Gabaix (2014). Another novelty of this paper is that CEO's effort is endogenous and interacts with both the choice of debt and the compensation contract proposed by the shareholder. These are key extensions for the results and policies that we study.

Our results complement the recent literature which analyzes executive compensation as

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<sup>1</sup>Krugman (1998) points to Michael Jackson's purchases of ski resorts during the 1997 Asian Financial crisis and how he mismanaged them. Aguiar and Gopinath (2005) and Acharya et al. (2011) also examine fire sales to inefficient investors.

<sup>2</sup>Otto (2014) and Humphery-Jenner et al. (2016) provide empirical support for this theory.

a policy tool. For example, John et al. (2000), Bebchuk and Spamann (2009), Bolton et al. (2015), Raviv and Sisk-Ciamarra (2013), Hakenes and Schnabel (2014), or Thanassoulis (2014). This literature has mostly focused on risk-shifting problems and externalities from competition in labor markets. Gete and Gómez (2015) compare compensation contracts in a model with overborrowing externalities but exogenous effort and exogenous compensation contracts. To our knowledge, this is the first paper to compare regulating compensation versus leverage regulations.

Finally, most of our results apply to recent representative agent models of overborrowing with collateral constraints in which the borrower does not internalize the link between her actions and asset prices, like Lorenzoni (2008), Bianchi (2010), Jeanne and Korinek (2010), or Stavrageva (2013). In those papers the agent does not internalize the right fire sale prices because she is small and ignores general equilibrium effects. In our model, the agent (the CEO) is optimistic. In any case, the agent overborrows because she underestimates the cost of fire sales.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 shows that shareholders propose compensation contracts that generate overleverage. Section 4 studies regulations to achieve social efficiency. Section 5 analyzes how our theoretical results may yield different empirical predictions across sectors. Section 6 concludes. The Appendix contains the algebra.

## 2 Model

There is a continuum of small firms which we model as a representative price-taker firm. The firm is composed of an executive (the CEO) and a shareholder. The shareholder owns the firm but the CEO manages it. Both are risk-neutral. The shareholder only decides the CEO's compensation contract. Besides the firm, there exists an unskilled investor. Next we discuss the setup and the problem of each agent.

### 2.1 Setup

There is one period and we denote its beginning and end by  $t_0$  and  $t_1$ . At the end of the period there are two states of nature (high and low) that we denote with superscripts,  $s = \{h, l\}$ .

There are two assets: a core asset and a new investment asset. The core asset represents the “steady-state strategy” of the company. It involves relatively low uncertainty. Thus, we model it as a risk-free asset that pays a deterministic gross return  $b > 0$ . At  $t_0$ , the firm is endowed with  $k$  units of the core asset. There exists as well a new investment asset that pays a gross return  $a^h$  if the state of nature is high, and  $a^l$  if it is low, with

$$a^l < R < a^h. \tag{1}$$

At  $t_0$ , the firm's CEO can borrow  $d \geq 0$  units at an interest  $R$  to invest in the investment asset. This investment asset represents a new, uncertain strategy. By putting some effort  $e > 0$  the CEO affects the likelihood of arriving to the high state-of-nature. That is, the high state occurs with endogenous probability  $p(e)$ , which is increasing in the effort exerted by the CEO. For example, this effort is the time and resources employed to search for new investment opportunities. Effort is not observable by the shareholder and the CEO's compensation cannot be contingent on it. This is the source of agency conflict in our model.<sup>3</sup> We assume that  $p(e) = e$ , and solve for the optimal effort directly as a function of  $p$ , that is

$$e(p) = p.$$

Providing effort is costly for the CEO with an increasing and convex cost function  $c(p)$ . We assume that there is a minimum effort level needed to run the firm:

$$p \geq \underline{p} = \frac{R - a^l}{a^h - a^l}, \quad (2)$$

with  $c'(\underline{p}) = 0$ . This assumption ensures that debt has a positive expected net payoff ruling out the trivial case in which  $d = 0$ .

At  $t_0$  the shareholder proposes a compensation contract to the CEO. The contract consists of a fixed salary  $F \geq 0$  and a variable payment that is a percentage  $\gamma \in [0, 1]$  of the firm's profit at the end of the period. If the CEO accepts the compensation contract, she must decide at  $t_0$  how much effort to exert and how much to borrow. If the CEO rejects the contract her reservation compensation is  $A$ .

At time  $t_1$  and state  $s$ , the CEO has to repay the debt and interests. If the return on the investment is not enough to repay, the CEO can sell part of the core assets,  $f^s$ , to the unskilled investor for a price  $q^s$ . As we show below, the purchase price the unskilled investor pays is below the value of the long-term asset,  $q^s < b$ , thus we refer to these sales as fire sales.

We denote the firm's profit at  $t_1$  as

$$\pi_1^s = b(k - f^s) + q^s f^s + (a^s - R) d.$$

We focus on non-default equilibria. That is, equilibria that satisfy the following non-negativity constraint:

$$q^s f^s + (a^s - R) d \geq 0. \quad (3)$$

In other words, we assume that, after the fire sales, all debt and interests are repaid. As explained below, this non-default constraint will help prevent losses in equilibrium.

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<sup>3</sup>Effort aversion has been studied by Jensen and Meckling (1976) and Harris and Raviv (1979), among many others.

## 2.2 The investor’s problem

At  $t_1$ , the investor can buy some of the core assets from the firm at price  $q^s$  per unit. We refer to the investor as “unskilled” because to manage  $x$  units of core assets she has to pay a quadratic cost  $\frac{1}{2}vx^2$ , with  $v \geq 0$ . The parameter  $v$  captures the marginal loss from early liquidation. Like in Krugman (1998), these costs are real costs which reduce total output; that is, they are not mere transfers across agents. For example, these costs can be inferior management or informational skills of the investor relative to the firm’s CEO.<sup>4</sup> Therefore, selling core assets is negative-NPV.

In equilibrium, by market clearing, the assets bought at  $t_1$  by the unskilled investor equal the assets sold by the firm’s CEO,  $f^s$ . The unskilled investor maximizes the value she would get from the assets purchased at price  $q^s$  net of purchase costs. That is,

$$\max_{f^s} \mathbb{E} \left[ bf^s - q^s f^s - \frac{1}{2}v (f^s)^2 \right],$$

subject to  $f^s \geq 0$ . The first-order-condition yields the price function

$$q^s \equiv q(f^s) = b - vf^s. \quad (4)$$

This is the price at which the investor would buy core assets from the CEO at  $t_1$ . It is decreasing in the volume of purchases because the cost of managing the assets increases in their volume. For positive sales, the price is always below the asset’s fundamental value,  $b$ , that is, the value if it remains managed by the firm.

## 2.3 The CEO’s problem

Like in Gabaix (2014), the CEO does not correctly internalize the price function (4). That is, she makes decisions at  $t_0$  assuming

$$q_m^s \equiv q_m(f^s) = b - mvf^s, \quad (5)$$

with  $m \in (0, 1]$ . For  $m < 1$  the CEO overvalues the asset prices at which she expects to sell in the low state of nature. Thus, we interpret  $m$  as a measure of the CEO’s optimism: smaller  $m$  would correspond to larger optimism.

Because the CEO would never sell at a negative price, fire sales will be limited to the range  $\bar{f}_m \geq f^s \geq 0$ , where  $\bar{f}_m$  satisfies  $q_m(\bar{f}_m) = 0$ , or  $\bar{f}_m = \frac{b}{mv}$ . We assume that, for any  $m$ , the parameters satisfy

$$\bar{f}_m \leq k. \quad (6)$$

Assumption (6) together with the non-default constraint (3) prevent the firm from entering into losses. Allowing firm losses is equivalent to removing assumptions (3), (6), and introducing

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<sup>4</sup>For example, the Financial Times (2012) reported that many funds buying mortgages from Spanish banks incurred significant costs to understand and assess their values.

restrictions on limited-liability for both the CEO and the shareholder. Payoffs will then exhibit a “kink.” It is well understood that limited liability may induce risk-shifting and overleverage (see, for instance, John et al. 2000). Our model generates overleverage through a different channel: managerial optimism.<sup>5</sup>

At the end of the period, the CEO’s expected discounted payments are

$$G(f^h, f^l, d, p) = F + \gamma V(f^h, f^l, d, p), \quad (7)$$

where

$$V(f^h, f^l, d, p) = p\pi_1^h + (1 - p)\pi_1^l, \quad (8)$$

is the firm’s expected profit at  $t_1$ .

The CEO takes as given the fixed and variable payments ( $F$  and  $\gamma$ ) and decides the level of debt and effort to maximize her expected discounted payments net of the effort cost:<sup>6</sup>

$$\max_{d, p, f^h, f^l} G(f^h, f^l, d, p) - c(p), \quad (9)$$

subject to (2), (3), the non-negative restrictions on  $f^s$  and  $d$ , and to her expected fire sales price function (5). Replacing  $f^h$ ,  $f^l$ , and  $d$  in the firm’s expected profit (8),  $V(f^h, f^l, d, p)$  becomes a function of  $p$  and  $m$  that we denote as  $V(p, m)$ .

## 2.4 The shareholder’s problem

At  $t_0$ , the shareholder proposes a compensation contract  $(F, \gamma)$  to maximize the firm’s expected profit net of the CEO’s compensation. Thus, he solves

$$\max_{F, \gamma} (1 - \gamma)V(p, m) - F,$$

subject to  $F \geq 0$ ,  $\gamma \in [0, 1]$ , the debt, effort and fire sales which solve the CEO’s problem, and to the CEO’s participation constraint

$$F + \gamma V(p, m) - c(p) \geq A. \quad (10)$$

## 3 Inefficient market equilibrium

First we identify the socially efficient allocations. Then we solve the CEO’s problem and characterize the compensation contract proposed by the shareholder. Managerial optimism

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<sup>5</sup>The non-default constraint (3) and condition (6) prevent the kink. These restrictions together with the assumption of risk neutrality make the model much more tractable and the intuitions more straight forward. On the other side, these assumptions will affect the shareholder’s optimal contract choice. We discuss the implications of these assumptions in Section 4.

<sup>6</sup>Because of (3), choosing the level of debt is equivalent to selecting the asset sales. As we will show below, it is never optimal to sell core assets in the high state of nature. In the low state the manager sells the core assets needed for (3) to be binding.

leads to socially inefficient overleverage. When effort is endogenous, leverage is shown to increase further due to the complementarity between CEO choices of leverage and effort. More importantly, the shareholder, even if he is not optimistic, has no incentive to correct the CEO. This result questions the efficacy of say-on-pay regulation to prevent excessive firm leverage.

### 3.1 Social efficiency

The social inefficiency is due to the resources the unskilled investor wastes when she acquires core assets ( $v > 0$ ). Pareto-efficient allocations optimize that waste of resources. Optimizing the waste does not eliminate such waste entirely, since such a result is only possible when debt is zero. But zero debt is not optimal since the expected return from debt is positive. The optimal leverage is achieved when the firm's CEO selects the right level of debt while correctly internalizing ( $m = 1$ ) the costs of the potential fire sales associated with debt. This is what we show in the next proposition:

**Proposition 1** *An allocation  $x = \{d, f^h, f^l, p\}$  is Pareto optimum if and only if whoever makes the leverage, effort and fire sales decisions internalizes the price function (4). That is, when  $m = 1$ .*

The intuition is that, in our model, the First Welfare Theorem fails because the CEO does not use the right price function (4). Optimism distorts the information content of prices, inducing the CEO to choose excessive debt and fire sales. Fire sales entail a real cost for society because  $v > 0$  is not a mere wealth transfer. A social planner could improve social welfare by reducing debt and the waste of resources in the low state of nature. The planner could then redistribute the gains from the Pareto efficient output to ensure everybody is better off.

### 3.2 CEO's choices

The following proposition characterizes the solution to the CEO's problem.

**Proposition 2** *For variable payments  $\gamma > 0$ , both debt and fire sales in the low state increase with effort  $p$  and with CEO's optimism (lower  $m$  means more optimism). That is,  $\frac{\partial d}{\partial m} < 0$ ,  $\frac{\partial f^l}{\partial m} < 0$ ,  $\frac{\partial d}{\partial p} > 0$ ,  $\frac{\partial f^l}{\partial p} > 0$ . The levels of debt and fire sales are:*

$$d = \frac{b^2}{(R - a^l) 4mv} \left( 1 - \left( \frac{(1 - p)(R - a^l)}{p(a^h - R)} \right)^2 \right), \quad (11)$$

$$f^h = 0, \text{ and } f^l = \frac{b}{2mv} \left( 1 - \frac{(1 - p)(R - a^l)}{p(a^h - R)} \right). \quad (12)$$

Effort,  $p(\gamma, m)$ , is implicitly defined by the incentive compatibility constraint,

$$\gamma \frac{\partial V(p, m)}{\partial p} = c'(p). \quad (13)$$



Effort increases with variable payments,  $\gamma$ , and decreases with  $m$ . Moreover,

$$\frac{\partial^2 p}{\partial m \partial \gamma} < 0. \quad (14)$$

When effort is endogenous, the total effect of optimism ( $m$ ) on borrowing,  $\frac{\partial d}{\partial m}$ , works through two channels that can be decomposed as follows:

$$\frac{\partial d}{\partial m} = \underbrace{\frac{\partial d}{\partial m} \Big|_p}_\text{Direct channel} + \underbrace{\frac{\partial d}{\partial p} \frac{\partial p}{\partial m}}_\text{Indirect channel}. \quad (15)$$

We denote by  $\frac{\partial d}{\partial m} \Big|_p$  the effect of  $m$  on  $d$  holding effort constant. This is the *direct channel* well known in models of managerial optimism: for a given fixed effort, higher optimism (smaller  $m$ ) leads to a larger overestimation of the revenues from fire sales and, ultimately, more debt.

When effort  $p$  is endogenous there is a second, *indirect channel* in (15). Through this new channel, endogenous effort reinforces overborrowing because there is a complementarity between effort and leverage,  $\frac{\partial d}{\partial p} > 0$ . Leverage is more profitable when effort is higher since more effort makes the high state of nature more likely. Moreover, effort is higher when the CEO overestimates the revenues from fire sales,  $\frac{\partial p}{\partial m} < 0$ . Therefore, CEO's optimism makes overborrowing larger when effort is endogenous than when it is exogenous, that is,  $\frac{\partial d}{\partial p} \frac{\partial p}{\partial m} < 0$ .

Result (14) says that more optimistic managers are more sensitive to compensation incentives. Hence, the effects of optimism on effort ( $\frac{\partial p}{\partial m} < 0$ ), and thus the importance of the *indirect channel* in (15), are larger the higher the variable payment.

We denote as  $p^*$  and  $f^{l*}$  the efficient choices of effort and fire sales corresponding to  $m = 1$ . An optimistic CEO ( $m < 1$ ) exerts an effort higher than the efficient level of effort,

$$p(\gamma, m) > p^*, \quad (16)$$

and overborrows:

$$d > d^* = \frac{b^2}{(R - a^l)4v} \left( 1 - \left( \frac{(1 - p^*)(R - a^l)}{p^*(a^h - R)} \right)^2 \right). \quad (17)$$

In the low state, the fire sales expected by the optimistic CEO are larger than the efficient level of fire sales:

$$f^l > f^{l*} = \frac{b}{2v} \left( 1 - \frac{(1 - p^*)(R - a^l)}{p^*(a^h - R)} \right), \quad (18)$$

but lower than the actual fire sales  $\hat{f}^l$  needed to avoid bankruptcy in the low state:

$$q(\hat{f}^l)\hat{f}^l + (a^l - R)d = 0.$$

The social inefficiency arises because the optimistic CEO expects that, in the low state of nature, she will sell  $f^l$  units at price  $q_m^l$  given by (5). Accordingly, she borrows  $d$  in (11). However, fire sales will take place at price  $q^l$  given by (4). Since the fire sale price is lower than expected by the CEO she ends up selling too many core assets ( $\hat{f}^l$ ) to avoid default which depresses fire sales prices even further.

### 3.3 Equilibrium Contract

The next proposition characterizes the contract selected by the shareholder. The CEO wants to be compensated up to her reservation salary. Optimism makes her underestimate the costs from asset sales, overestimate firm's profits and thus accept a lower share of them as compensation ( $\frac{\partial \gamma}{\partial m} > 0$ ).

**Proposition 3** *For a given  $m \leq 1$ , the shareholder offers a contract with no fixed salary ( $F = 0$ ) and a percentage of the firm's profit ( $\gamma < 1$ ) which is smaller the larger the CEO's optimism ( $m$  smaller):*

$$\frac{\partial \gamma}{\partial m} > 0. \quad (19)$$

*The CEO's participation constraint is binding at her reservation compensation,  $A$ .*

In the next proposition we show that the shareholder, even if he is rational, has no incentive to correct the CEO's inefficient overleverage.<sup>7</sup>

**Proposition 4** *The shareholder's expected profit at  $t_1$  net of the executive compensation, is  $V(p, m) - c(p) - A$ . The variation of this net profit with respect to the CEO's optimism can be written as*

$$\frac{\partial (V(p, m) - c(p) - A)}{\partial m} = \underbrace{\frac{\partial \hat{V}(p, m)}{\partial m} \Big|_p}_{(> 0) \text{ Social cost of optimism}} + \underbrace{\frac{\partial \Delta(p, m)}{\partial m} \Big|_p}_{(< 0) \text{ Wealth transfer}} + \underbrace{\frac{\partial [V(p, m) - c(p)]}{\partial p} \frac{\partial p}{\partial m}}_{(< 0) \text{ Enhanced effort}} < 0, \quad (20)$$

where  $\frac{\partial \hat{V}(p, m)}{\partial m} \Big|_p$  denotes the variation of  $\hat{V}(p, m)$  with respect to  $m$  when  $p$  is constant. We use similar notation for  $\frac{\partial \Delta(p, m)}{\partial m} \Big|_p$ .  $\hat{V}(p, m)$  is the firm's expected profit at the actual level of fire sales in the low state:

$$\hat{V}(p, m) = p((a^h - R)d + bk) + (1 - p)b(k - \hat{f}^l). \quad (21)$$

$\Delta$  represents the optimistic CEO's overestimation of the firm's profit at  $t_1$ ,

$$\Delta(p, m) = V(p, m) - \hat{V}(p, m) = (1 - p)b(\hat{f}^l - f^l) > 0. \quad (22)$$

Proposition 4 shows that the shareholder proposes a compensation package that optimizes his return but is socially inefficient. Equation (20) decomposes the result into three components.

The first component in the right hand side of (20) is what we call *social cost of optimism*. For a given level of effort, optimism ( $m < 1$ ) results into overleverage (Proposition 2) and, in

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<sup>7</sup>By rational we mean that the shareholder is aware that fire sales prices are actually determined by (4) and not by (5).

the case of the low state, excessive fire sales. This erodes the firm’s profits and the shareholder’s net payoff; that is,  $\frac{\partial \hat{V}(p,m)}{\partial m} > 0$ .

The second component in the right hand side of (20) is what we call the *wealth transfer* from the CEO to the shareholder,  $\Delta$ . An optimistic CEO, because she overestimates the firm’s profits, “saves” shareholders part of her compensation. We refer to this as a wealth transfer to the shareholder.<sup>8</sup> Notice that this component and the first component would arise even if effort were exogenous.

The third and last component in the right hand side of (20) is what we call the *enhanced effort* channel. Optimism encourages the CEO to leverage more and provide more effort (Proposition 2). This is valuable to the shareholder because higher effort reduces the probability of the low state of nature. This component arises only when effort is endogenously determined.

The first component of (20) induces the shareholder to correct the CEO. However, the second and third components show that, because of the unpaid extra effort that optimistic CEOs provide, the shareholder is better off by letting the CEO overleverage.

## 4 Regulation

In the previous section we showed that when CEOs are optimistic, the unregulated market equilibrium is inefficient. The CEO overborrows and the shareholder, even if he is not optimistic, has no incentive to correct the CEO. In this section we analyze two tools to induce social efficiency. First, we restrict shareholders’ choices on the structure (although not the level) of executive compensation. That is, we impose a cap on the variable relative to the fixed salary. Second, we directly regulate the leverage level, like with standard capital requirements or the leverage restrictions discussed by Korinek and Jeanne (2014). Finally, we compare the two policy tools.

### 4.1 Regulating executive compensation

The regulator imposes a cap  $T$  on the ratio of variable-to-fixed CEO’s compensation,<sup>9</sup>

$$T \geq \frac{\gamma V(p, m)}{F}. \quad (23)$$

The shareholder is constrained by the cap when he designs the CEO’s compensation and alters the compensation contract:

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<sup>8</sup>At  $t_0$ , the CEO based her decisions on the price function (5) and agreed to work in exchange for her reservation compensation plus her effort cost. However, if the low state arrives, it is the price function (4) that governs asset prices. Asset prices are lower than expected by the optimistic CEO, thus the actual fire sales will be larger than expected ( $\hat{f}^l > f^l$ ) and profits and payments to the CEO are smaller. The CEO is ultimately paid less than her reservation utility.

<sup>9</sup>We use the subscript  $T$  to denote the solutions to this restricted problem.

**Proposition 5** *The shareholder proposes a contract in which the cap constraint (23) is binding. The variable payments increase as the cap is relaxed,  $\frac{\partial \gamma_T}{\partial T} > 0$ . The fixed salary is*

$$F_T = \frac{A + c(p_T)}{(1 + T)} > 0. \quad (24)$$

Effort is a function of  $\gamma$  via (13). As the cap becomes tighter (smaller  $T$ ), the variable share decreases and the CEO has less incentive to provide effort ( $\frac{\partial p_T}{\partial T} > 0$ ). Lower effort leads to lower leverage and fewer fire sales in the low state of nature.

It is important to stress that there is a tradeoff between achieving the socially efficient level of debt,  $d^*$ , defined in (17) and the efficient amount of effort,  $p^*$ , defined in (16). For any  $m < 1$ , achieving  $d^*$  implies an effort provision lower than the socially efficient level  $p^*$ . On the other side, inducing  $p^*$  implies inefficient overleverage  $d_T > d^*$ . This tradeoff exists because the cap  $T$  reduces variable payments and this discourages the executive from providing effort.

Figure 1 illustrates this tradeoff numerically.<sup>10</sup> The figure plots in the x-axis different levels of the cap  $T$ . In the y-axis, the top panel plots the CEO's debt choice while the bottom panel plots her effort level. Both panels include the efficient levels of debt and effort ( $d^*$  and  $p^*$ , respectively).

[Insert Figure 1 around here]

Figure 1 shows that as the cap is tighter ( $T$  smaller) the regulator prevents the shareholder from providing too much variable pay. As a consequence the CEO exerts less effort and borrows less. The complementarity between effort and leverage implies that as debt moves towards the optimal  $d^*$  in the top panel then effort becomes smaller than the efficient level  $p^*$  in the bottom panel. In other words, to lower leverage regulators need to lower the share of variable pay, which ultimately disincentivizes effort provision.

Alternatively, if the regulator is targeting a socially efficient provision of effort  $p^*$  in the bottom panel, the top panel shows that the corresponding cap on variable compensation leads the CEO to overleverage relative to the efficient level of debt  $d^*$ .

## 4.2 Regulating leverage

We assume in this section that the CEO solves the problem (7) subject to the same restrictions plus an additional restriction on debt imposed by the regulators.<sup>11</sup>

<sup>10</sup>We assume the following parameter values for all figures:

Parameters			
$a^h = 1.1$	$a^l = 0.9$	$R = 1$	$A = 0.05$
$v = 1$	$k = 5.25$	$b = 1.05$	

Since the marginal utility of effort is increasing in  $p$ , we use a functional form convex enough to ensure an interior solution for effort:  $c(p) = 6(p - p)^3$ .

<sup>11</sup>We use the subscript  $d$  to denote the solutions to this restricted problem. The proofs in this section follow immediately from the proof of Proposition 2 after including the debt constraint (25).

$$d \leq \bar{d}. \quad (25)$$

When the debt constraint is binding,<sup>12</sup> the CEO chooses debt  $\bar{d}$ , fire sales  $f_d^h = 0$  in the state  $s = h$ , and

$$f_d^l(\bar{d}) = \frac{b}{2mv} \left( 1 - \sqrt{1 - \frac{(R - a^l)4mv}{b^2} \bar{d}} \right), \quad (26)$$

in the state  $s = l$ . Effort  $p(\gamma, \bar{d}, m)$  is implicitly defined by the incentive compatibility constraint,

$$\gamma \frac{\partial V(p, \bar{d}, m)}{\partial p} = c'(p). \quad (27)$$

where  $V(p, \bar{d}, m)$  is the expected profit function:

$$V(p, \bar{d}, m) = bk + \bar{d}(p(a^h - R) - (1 - p)(R - a^l)) - (1 - p)mv (f_d^l(\bar{d}))^2.$$

The optimal contract implies  $F_d = 0$  and a share  $\gamma_d$  of profits such that the participation constraint is binding:

$$\gamma_d V(p_d, \bar{d}, m) = c(p_d) + A. \quad (28)$$

Constraints (27) and (28) jointly determine  $\gamma_d$  and  $p_d = p(\gamma_d, \bar{d}, m)$ .

[Insert Figure 2 around here]

Figure 2 displays the levels of debt and effort for different levels of the cap on leverage. If the cap is too loose the CEO overborrows and provides excessive effort.

### 4.3 Comparing regulations

A natural question is whether, from the point of view of the regulator, any of the two tools (a cap on variable pay or a cap on debt) is preferable. The following proposition shows that a cap on debt is a better policy tool because it can achieve the socially efficient level of debt with a higher provision of effort.

**Proposition 6** *Given a cap on the variable compensation  $T^*$  that yields the socially efficient level of debt  $d^*$ , the CEO's optimal provision of effort,  $p_{T^*}$ , is lower than the effort  $p_{d^*}$  exerted by the same CEO when the cap on variable compensation is replaced with a cap  $\bar{d} = d^*$  on debt. Moreover, the variable compensation in the later case,  $\gamma_{d^*}$  is higher than the variable compensation in the former,  $\gamma_{T^*}$ .*

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<sup>12</sup>If the leverage constraint (25) is not binding, the CEO's optimal choice is characterized in Proposition 2 and the contract in Proposition 3.

Both policy tools can achieve the optimal level of debt. However, the cap on variable pay leads to a higher distortion in the provision of effort. It imposes a fixed salary higher than what shareholders would choose if they were free to select the compensation contract that delivers the socially optimal level of debt. Since higher fixed salary discourages effort, the cap on variable compensation makes the low state of nature (a crisis) more likely than the cap on leverage.<sup>13</sup>

[Insert Figure 3 around here]

Figure 3 confirms our previous result. When caps are set such that both policy tools induce the socially optimal debt level  $d^*$ , the cap on debt (dotted line, right scale) induces higher effort and variable compensation than the cap on compensation (dashed line, left scale). In other words, if the level of debt measures the size of a crisis, then both tools ensure crises of the same size. However, regulating compensation makes crises more likely.

It is worth discussing now the generality of Proposition 6 in the light of the simplifying assumptions that we have made to render the model more tractable. Restrictions (3) and (6) prevent the firm to enter in losses at  $t_1$  in the low state. As a consequence, the limited-liability restriction will never be binding. Removing these restrictions and allowing firm losses may result in an optimal contract with positive fixed payments ( $F > 0$ ) necessary to meet the limited-liability constraint. This may be the case even when there exists a cap on debt. The assumption of risk-neutrality implies that the CEO needs not be compensated for the extra risk she assumes when she is induced to expend costly effort by the shareholder. This is captured by the incentive compatibility constraint (13) or, alternatively, the constraint (27) if debt is restricted. If the CEO is risk-averse, a risk-premium must be paid. This will likely imply a positive fixed payment necessary to meet the CEO's (binding) participation constraint. Hence, removing our simplifying assumptions will likely result in a positive fixed payment when debt is restricted.

Heuristically, the intuition in Proposition 6 should hold as long as the fixed payment necessary to attain the socially optimal leverage ( $F_{T^*}$ ) when the variable bonus is capped is higher than the shareholder's optimal fixed salary under the leverage cap ( $F_{d^*}$ ). Theoretically, this will depend, among other things, on the CEO's risk-aversion, her disutility of effort and her reservation salary.

## 5 Empirical predictions

The model generates a number of empirical predictions involving firm leverage (defined as assets to equity), executive's effort and compensation. We outline the predictions below.

First, optimism is positively associated with leverage and effort, and this effect is larger

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<sup>13</sup>Proposition 6 shows that the leverage restriction is better than the cap on variable compensation because it achieves the optimal debt level  $d^*$  and gets closer to the optimal effort level. This does not necessarily mean that the planner would choose that leverage restriction, because the planner would optimize over the entire  $(d, p)$  space.

in sectors (firms) in which the executive’s effort plays a relevant role in the success of any investment. This includes, for example, sectors in which executives’ soft skills and information acquisition are key for the investment success. Among these sectors (firms), those with higher leverage should have their executives exerting higher effort ( $\frac{\partial d}{\partial p} > 0$ ). There has been some work on the first part of the prediction, but not on the other components. For example, Ben-David et al. (2013) find some evidence that firms with optimistic executives invest more and have more debt. Malmendier et al. (2011) find that optimistic managers use leverage more aggressively. The long hours usually associated with the financial industry may be anecdotal evidence for the complementarity between effort and leverage.

Second, Otto (2014) finds that firms profit from the overconfidence of CEOs who overestimate the future value of the firm’s equity by granting fewer stock options and lower bonuses. Our theory adds a cross-sectional dimension: shareholders will take advantage of this feature especially in sectors in which effort is less observable and leverage is higher, like the financial industry. In these two cases Proposition 4 shows that the gains for shareholders from the unpaid enhanced effort are larger.

Third, there is a growing empirical literature showing that asset booms and leverage are positively correlated (see for example, Jorda et al. 2013). If we assume that optimism is more likely in periods of asset booms then our model predicts that episodes of rapid increases in corporate leverage (like the recent experiences of emerging markets) are associated with increases in the variable share of compensation. This is because optimistic executives overvalue the variable pay and shareholders may have no incentives to undo this bias as we showed in Proposition 4. Moreover, our theory would predict that if some countries favor variable compensation more than others (for example, different fiscal treatments) the elasticity of leverage growth to asset price growth would be larger. This may be of interest for cross-country studies linking leverage and asset prices, like Giacomini et al. (2014).

Finally, our model predicts that say-on-pay regulation will not help in mitigating leverage in periods of asset booms and optimism. In fact, say-on-pay can reinforce overborrowing if shareholders design contracts to profit from optimistic CEOs. This may lead to testable predictions comparing countries with different say-on-pay regimes. Similarly, proposition 6 suggests that imposing a cap on variable compensation distorts effort more than regulating leverage. Empirical work could analyze whether different regulations of executive compensation alter the frequency of firms’ fire-sales or defaults across countries.

## 6 Conclusions

In this paper we have analyzed a model with endogenous determination of leverage, executive compensation and CEO’s effort. Overborrowing arises due to CEO’s optimism. Our insights come from making the CEO’s effort endogenous and non-contractible.

Our results show that when executives are optimistic about asset prices in states of distress, shareholders propose compensation packages that lead to socially excessive leverage. This result provides support for regulation and suggests that say-on-pay regimes may induce greater

leverage. This result may motivate further empirical work because Correa and Lel (2016) show that say-on-pay laws have lead to substantial changes in executive compensation.

We find that, at least for risk-neutral agents, the optimal regulation is not the regulation of executive compensation. A cap on debt is socially more efficient: it can restore the efficient level of debt with a lower distortion in managerial effort. In any case, decreasing leverage reduces the losses of financial distress, but simultaneously weakens the incentives (i.e. effort) necessary to make crises less likely.



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# Appendix

## Proof of Proposition 1

The proof is similar to showing that the First Welfare Theorem fails when one agent uses distorted prices. Optimism is distorting the prices used by the CEO. To trace the Pareto Frontier of efficient allocations, we solve the problem of a planner who chooses the efficient allocation of production among the set of feasible allocations and then redistributes the output among the agents using lump-sum taxes or transfers  $(T^s, \tilde{T}^s)$  in zero net supply.

By definition, the payments to the shareholder and to the CEO must add up to firm's profits. We can define the firm's expected profit net of transfers and effort cost as

$$U_B = E(\pi_1^s + T^s - c(p)). \quad (\text{A1})$$

The expected profit of the unskilled investor net of transfers is defined as

$$U_U = E\left(bf^s - q^s f^s - \frac{1}{2}v(f^s)^2 + \tilde{T}^s\right). \quad (\text{A2})$$

The transfers must be in zero net supply:

$$T_t^s + \tilde{T}_t^s = 0, \quad \forall t, \forall s. \quad (\text{A3})$$

**Definition 1** *The set of feasible allocations is the set  $F = \{d, f^h, f^l, p\}$  such that the following equations hold: (2), (3),  $d > 0$ , and market clearing in asset sales.*

**Definition 2**  *$P \subset F$  denotes the set of Pareto allocations. That is, for all allocations  $x = \{d, f^h, f^l, p\} \in P$  there is no other allocation  $x' \in F$  for which there exists a system of transfers  $\{T_t^s, \tilde{T}_t^s\}$  satisfying (A3) such that  $U_B(x') \geq U_B(x)$ ,  $U_U(x') \geq U_U(x)$  with at least one the previous inequalities being strict inequality.*

The planner problem traces the Pareto Frontier when maximizing a weighted sum of the expected profits of firms and unskilled investors among the allocations in the feasibility set  $F$ . Denoting the social weight of the unskilled investor as  $1 \geq \Psi \geq 0$ , the social planner solves for

$$U = \max_{d,p,\{f^s,T_t^s,\tilde{T}_t^s\}_{s=h,l}} \left\{ (1 - \Psi)U_B(d, f^s, T_t^s) + \Psi U_U(\tilde{f}^s, \tilde{T}_t^s) \right\}, \quad (\text{A4})$$

subject to  $\{d, f^h, f^l, p\} \in F$  and to the zero-net supply transfers (A3).

The set of FOCs from problem (A4) includes the price function (4). Thus, any allocation decided by the CEO using (5) with  $m < 1$  leads to a suboptimal level of fire sales, and to lower output because the costs paid by the unskilled investor are wasted resources. Those allocations cannot be Pareto efficient because, for any weight  $\Psi$ , the planner could always choose an allocation solving her problem (thus using the price function (4)). The planner's allocation will have higher total output by definition of the Pareto frontier. Then the planner can redistribute the higher output to make everybody better off. In other words, the First Welfare Theorem applies to our economy when the agents use the right prices.

## Proof of Proposition 2:

We remove the subscript  $m$  to simplify the notation. For  $s = \{l, h\}$ , the CEO solves  $\max_{f^h, f^l, d, p} G(f^h, f^l, d, p) - c(p)$  subject to (3),  $\bar{f} \geq f^s \geq 0$ ,  $d \geq 0$ ,  $p \geq \underline{p}$ , and  $q^s(f^s) = b - mvf^s$ . We define the Lagrangian function  $L(f^h, f^l, d, p) = G(f^h, f^l, d, p) - c(p) + \lambda^h(f^h q^h + (a^h - R)d) + \lambda^l(f^l q^l + (a^l - R)d) + \varphi^h(\bar{f} - f^h) + \varphi^l(\bar{f} - f^l) + \psi^h f^h + \psi^l f^l + \rho d + \phi(p - \underline{p})$ , with the non-negative Lagrange multipliers  $\lambda^s$ ,  $\psi^s$ ,  $\rho$ , and  $\phi$ . In addition, the following slackness conditions must hold:  $\lambda^h(f^h q^h + (a^h - R)d) = 0$ ,  $\lambda^l(f^l q^l + (a^l - R)d) = 0$ ,  $\varphi^h(\bar{f} - f^h) = 0$ ,  $\varphi^l(\bar{f} - f^l) = 0$ ,  $\psi^h f^h = 0$ ,  $\psi^l f^l = 0$ ,  $\rho d = 0$ ,  $\phi(p - \underline{p}) = 0$ . The FOCs are:

$$\lambda^h b - 2mvf^h(p\gamma + \lambda^h) + \psi^h = \varphi^h, \quad (\text{A5})$$

$$\lambda^l b - 2mvf^l((1-p)\gamma + \lambda^l) + \psi^l = \varphi^l, \quad (\text{A6})$$

$$\rho + (p\gamma + \lambda^h)(a^h - R) = ((1-p)\gamma + \lambda^l)(R - a^l), \quad (\text{A7})$$

$$\frac{\partial G(f^h, f^l, d, p)}{\partial p} + \phi = c'(p). \quad (\text{A8})$$

First, we analyze fire sales in the high state,  $f^h$ . Assume  $d > 0$  (to be proved below). The slackness conditions and the assumption  $(a^h - R) > 0$  imply  $\lambda^h = 0$  and  $\rho = 0$ . By the same conditions, an interior solution ( $\bar{f} > f^h > 0$ ) would imply  $\psi^h = \varphi^h = 0$ . Then, from (A5), it follows that  $f^h = 0$ .

We turn now to the fire sales in the low state,  $f^l$ . Assuming again  $d > 0$ , and given  $(R - a^l) > 0$ , in the low state of nature the CEO needs to sell  $f^l > 0$  to cover debt and interest payments. By the slackness conditions,  $f^l > 0$  leads to  $\psi^l = 0$ . Then, (A7) implies  $\lambda^l = \gamma(1-p) \left( \frac{p(a^h - R)}{(1-p)(R - a^l)} - 1 \right) \geq 0$ , which holds with equality for  $p = \underline{p}$ . Now we prove that the non-negativity constraint (3) is binding in the low state. Assume it is not binding. Then  $\lambda^l = 0$  because of the slackness conditions. Given (A7), if  $p > \underline{p}$ , the multiplier  $\rho = \gamma((1-p)(R - a^l) - p(a^h - R)) < 0$ , which contradicts the non-negativity assumption of the multipliers. Thus, for  $p > \underline{p}$ ,  $f^l q^l + (a^l - R)d = 0$  and we obtain the level of debt characterized in Proposition 2. We prove now that  $f^l < \bar{f}$ . Assume  $f = \bar{f}$ . Then,  $\psi^l = 0$ , by the slackness conditions. Replacing  $\lambda^l$  in (A6), it follows that  $\varphi^l = -(1-p)\gamma \left( \frac{p(a^h - R)}{(1-p)(R - a^l)} + 1 \right) b < 0$ . This contradicts the non-negativity assumption of the multipliers. Hence,  $f^l < \bar{f}$  and, by the slackness conditions,  $\varphi^l = 0$ . Replacing  $\lambda^l$  in (A6), it follows that  $f^l$  is positive if and only if  $p > \bar{p}$ . Therefore, given  $d = \frac{q^l f^l}{(R - a^l)}$ , condition (2) is necessary and sufficient for  $d > 0$  and  $\frac{\partial G(f^h, f^l, d, p)}{\partial p} > 0$ . Since we have assumed  $c'(\underline{p}) = 0$ , then (A8) implies that  $p > \underline{p}$ , and the slackness condition implies  $\phi = 0$ .

Replacing  $f^h$ ,  $f^l$ , and  $d$  in the firm's expected profit,  $V(f^h, f^l, d, p)$ , we can write it as a function of  $p$  and  $m$ :

$$V(p, m) = bk + p \frac{b^2}{4mv} \frac{(a^h - R)}{(R - a^l)} \left( 1 - \frac{(1-p)(R - a^l)}{p(a^h - R)} \right)^2 > 0. \quad (\text{A9})$$

Thus, we can write (A8) as follows:

$$\gamma \frac{b^2}{4mv} \frac{R - a^l}{a^h - R} \left( \left( 1 + \frac{a^h - R}{R - a^l} \right)^2 - \frac{1}{p^2} \right) = c'(p). \quad (\text{A10})$$

The left-hand-side of (A10) is the derivative of the CEO's variable payments relative to her effort, that we denote by  $\gamma \frac{\partial V(p, m)}{\partial p}$ . A sufficient condition for the solution to the CEO's problem to be a local maximum is that the Lagrangian function evaluated at the optimal is negative semidefinite. This condition requires all the first principal minors of the Hessian matrix for the Lagrangian function to be non-positive. We assume that the inequality is strict:

$$\gamma \frac{\partial^2 V(p, m)}{\partial p^2} - c''(p) < 0. \quad (\text{A11})$$

By the Implicit Function Theorem, taking the derivative of (A10) with respect to  $m$  and solving for  $\frac{\partial p}{\partial m}$  we obtain  $\frac{\partial p}{\partial m} = -\gamma \frac{\partial^2 V}{\partial p \partial m} \left( \gamma \frac{\partial^2 V}{\partial p^2} - c''(p) \right)^{-1}$ . The result  $\frac{\partial p}{\partial m} < 0$  follows from  $\frac{\partial^2 V}{\partial p \partial m} < 0$  and (A11). Taking the derivative of  $\frac{\partial p}{\partial m}$  with respect to  $\gamma$ , and given the signs of the the partial derivatives, it is immediate to prove (14).

Likewise, the derivative of effort with respect to  $\gamma$  can be implicitly derived from the CEO's incentive compatibility condition (A10):  $\frac{\partial p}{\partial \gamma} = -\frac{\partial V(p, m)}{\partial p} \left( \gamma \frac{\partial^2 V}{\partial p^2} - c''(p) \right)^{-1} > 0$ . The inequality follows from  $\frac{\partial V(p, m)}{\partial p} > 0$ , and (A11).

### Proof of Proposition 3

The shareholder proposes a contract  $(F, \gamma)$  that maximizes her revenue,

$$\max_{F, \gamma} (1 - \gamma)V(p, m) - F \quad (\text{A12})$$

subject to  $\gamma \frac{\partial}{\partial p} V(p, m) = c'(p)$ ,  $F + \gamma V(p, m) - c(p) \geq A$ ,  $\gamma \geq 0$ ,  $\gamma \leq 1$ , and  $F \geq 0$ . The corresponding Lagrangian is:  $L(F, \gamma) = (1 - \gamma)V(p, m) - F + \lambda_1(\gamma \frac{\partial}{\partial p} V(p, m) - c'(p)) + \lambda_2(F + \gamma V(p, m) - c(p) - A) + \lambda_3 \gamma + \lambda_4(1 - \gamma) + \lambda_5 F$ . The non-negative multipliers are  $\lambda_1$  to  $\lambda_5$ . The following slackness conditions must hold:  $\lambda_2(F + \gamma V(p, m) - c(p) - A) = 0$ ,  $\lambda_3 \gamma = 0$ ,  $\lambda_4(1 - \gamma) = 0$ ,  $\lambda_5 F = 0$ . The FOCs are:

$$\lambda_2 - 1 + \lambda_5 = 0, \quad (\text{A13})$$

$$\frac{\partial V(p, m)}{\partial p} \frac{\partial p}{\partial \gamma} (1 - \gamma) + (\lambda_2 - 1)V(p, m) + \lambda_3 - \lambda_4 = 0. \quad (\text{A14})$$

We can show that it is optimal for the shareholder to propose  $F = 0$  and  $\gamma < 1$ . First, by contradiction we prove that  $F > 0$  and  $0 < \gamma < 1$  cannot be a solution. Assume  $F > 0$  and  $0 < \gamma < 1$ . Then  $\lambda_3 = \lambda_4 = \lambda_5 = 0$  by the slackness conditions, and  $\lambda_2 = 1$  by (A13). Then, given (A14),  $\frac{\partial V(p,m)}{\partial p} \frac{\partial p}{\partial \gamma} (1 - \gamma) = 0$ , which can only be true for  $\gamma = 1$ . We show now that  $F = 0$  and  $0 < \gamma < 1$  is a solution. If we assume so, then, by the slackness conditions,  $\lambda_3 = \lambda_4 = 0$ . From (A13) and (A14),  $\frac{\partial V(p,m)}{\partial p} \frac{\partial p}{\partial \gamma} (1 - \gamma) = \lambda_5 V(p, m)$ , which holds if and only if  $\lambda_5 > 0$ . This is consistent with  $F = 0$ . Finally, given (A9),  $(1 - \gamma)V(p, m) > 0$  for any  $\gamma < 1$ . This rules out  $F \geq 0$  and  $\gamma = 1$  as a solution. Therefore, at the optimal,  $F = 0$  and  $0 < \gamma < 1$ .

The participation constraint is binding:

$$\gamma V(p, m) = c(p) + A. \quad (\text{A15})$$

Given the incentive compatibility constraint (13), it is suboptimal to pay the CEO any compensation above her reservation utility net of the cost of effort. It would not increase the CEO's effort and it would decrease the shareholder's net profit. The optimal variable payment  $\gamma$  and effort  $p$  are jointly determined by the incentive and by the participation constraints.

Taking the total derivative of  $\gamma V(p, m) - c(p) - A = 0$  with respect to  $m$  and using the incentive compatibility constraint (13), it follows that

$$\frac{\partial \gamma}{\partial m} = -\gamma \frac{\partial V(p, m)}{\partial m} (V(p, m))^{-1} > 0.$$

The inequality follows from  $V(p, m)$  being positive and decreasing in  $m$ .

## Proof of Proposition 4

The shareholder's expected profit net of the CEO's compensation is  $(1 - \gamma)V(p, m) - F$ . If the CEO's participation constraint is binding,  $\gamma V(p, m) + F = A + c(p)$ . Replacing the later in the former, the shareholder's net profit becomes  $V(p, m) - c(p) - A$ . Taking the derivative of the shareholder's profit with respect to  $m$  we obtain

$$\frac{\partial(V(p, m) - c(p) - A)}{\partial m} = \frac{\partial V(p, m)}{\partial m} \Big|_p + \frac{\partial(V(p, m) - c(p))}{\partial p} \frac{\partial p}{\partial m} < 0. \quad (\text{A16})$$

The first term in (A16) represents the variation of  $V(p, m)$  with respect to  $m$  when  $p$  is constant. It is negative,  $\frac{\partial V(p, m)}{\partial m} \Big|_p < 0$ , because of (A9). From the FOC (13) and Proposition 3,  $\gamma = c'(p) \left( \frac{\partial V(p, m)}{\partial p} \right)^{-1} < 1$ . Hence,  $\frac{\partial(V(p, m) - c(p))}{\partial p} > 0$ . Finally, from Proposition 3,  $\frac{\partial p}{\partial m} < 0$ .

The actual fire sales in the low state,  $\hat{f}^l$ , are calculated such that  $q(\hat{f}^l)\hat{f}^l + (a^l - R)d = 0$ . Replacing  $d$  from Proposition 2 in the later equation we obtain

$$\hat{f}^l = \frac{b}{2v} \left( 1 - \sqrt{\frac{1}{m} \left( \left( \frac{(1-p)(R-a^l)}{p(a^h-R)} \right)^2 - (1-m) \right)} \right). \quad (\text{A17})$$

Substituting (A17) into  $V(p, m)$ , we obtain equation (21). The inequality (20) follows after replacing (21) and (22) into (A16).

Taking the derivative of (21) with respect to  $m$  we obtain

$$\frac{\partial \hat{V}(p, m)}{\partial m} \Big|_p = \frac{b}{4vm^2} \left( 1 - \left( \frac{(1-p)(R-a^l)}{p(a^h-R)} \right)^2 \right) \frac{p(a^h-R)}{R-a^l} \left( \frac{\sqrt{\left( \frac{(1-p)(R-a^l)}{p(a^h-R)} \right)^2 m}}{\sqrt{\left( \frac{(1-p)(R-a^l)}{p(a^h-R)} \right)^2 + (m-1)}} - 1 \right).$$

This expression is strictly positive for all  $m < 1$  if and only if  $\left( \frac{(1-p)(R-a^l)}{p(a^h-R)} \right)^2 (m-1) > (m-1)$ .

This is equivalent to  $\frac{(1-p)(R-a^l)}{p(a^h-R)} < 1$ , which follows from parameter restriction (2). Moreover,  $\frac{\partial \hat{V}(p, m)}{\partial m} \Big|_p = 0$  for  $m = 1$ . Finally, for  $m = 1$  we have  $\frac{\partial \hat{f}^l}{\partial m} < 0$  and  $\hat{f}^l = f^l$ . Hence,  $\hat{f}^l > f^l$  for all  $m < 1$ . Given (22), it follows that  $\frac{\partial \Delta}{\partial m} \Big|_p < 0$ .

## Proof of Proposition 5

The shareholder solves the same problem as in (A12) but replacing the non negativity constraint on  $F$  with the condition  $TF \geq \gamma V(p, m)$ . The Lagrangian is defined as before. The last slackness condition becomes  $\lambda_5(TF - \gamma V(p, m)) = 0$ . The FOCs with respect to  $F$  and  $\gamma$  are:

$$\lambda_2 - 1 + T\lambda_5 = 0, \quad (\text{A18})$$

$$\frac{\partial V(p, m)}{\partial p} \frac{\partial p}{\partial \gamma} (1 - (1 + \lambda_5)\gamma) + (\lambda_2 - 1 - \lambda_5)V(p, m) + \lambda_3 - \lambda_4 = 0. \quad (\text{A19})$$

As it was shown in the proof of Proposition 3,  $\gamma_T > 0$  and  $\lambda_3 = 0$ . By contradiction, we show now that  $\gamma_T < 1$ . Assume  $\gamma_T = 1$ . Then (A19) implies that  $-\frac{\partial V(p, m)}{\partial p} \frac{\partial p}{\partial \gamma} \lambda_5 + (\lambda_2 - 1 - \lambda_5)V(p, m) - \lambda_4 = 0$ . Given (A18), a necessary condition for this equality is  $\lambda_5 < \lambda_2 - 1 < 0$ . This contradicts the non-negativity condition on the multipliers. Hence,  $\gamma_T < 1$  and, by the corresponding slack condition,  $\lambda_4 = 0$ .

We prove by contradiction that  $\lambda_5 > 0$ . Assume  $\lambda_5 = 0$ . By (A18),  $\lambda_2 = 1$ . Replacing these values in (A19) the FOC becomes  $\frac{\partial V(p, m)}{\partial p} \frac{\partial p}{\partial \gamma} (1 - \gamma) = \lambda_4$ . Since  $\gamma_T < 1$  the FOC implies  $\lambda_4 > 0$  which contradicts the slackness condition. Hence,  $\lambda_5 > 0$ . This implies:

$$\gamma_T V(p_T, m) = TF_T. \quad (\text{A20})$$

Assume  $\lambda_2 > 0$ . Then (24) follows from replacing (A20) in the binding participation constraint. Replacing (24) in the binding slackness condition (A20), the optimal  $\gamma$  must satisfy:

$$\gamma_T V(p_T, m) = \frac{T}{1+T} (c(p_T) + A). \quad (\text{A21})$$

From (A21),  $\frac{\partial \gamma_T}{\partial T} = \frac{c(p_T) + A}{(1+T)^2} \left( V(p_T) + \frac{c'(p_T)}{1+T} \frac{\partial p_T}{\partial \gamma} \right)^{-1} > 0$ . Hence, the optimal variable payment incentive increases as the cap constraint on the variable variable payment is relaxed.

## Proof of Proposition 6

We define the cap on variable compensation  $T^*$  such that the CEO chooses an amount of debt equal to  $d^*$ . In other words, comparing (11) and (17),  $p_{T^*}$  is such that

$$1 - \left( \frac{(1 - p^*)(R - a^l)}{p^*(a^h - R)} \right)^2 = \frac{1}{m} \left( 1 - \left( \frac{(1 - p_{T^*})(R - a^l)}{p_{T^*}(a^h - R)} \right)^2 \right).$$

To induce effort  $p_{T^*}$ ,  $T^*$  is chosen such that the incentive compatibility constraint (13) and the slack condition (A21) are jointly satisfied. This, together with (24), yields the optimal contract  $(F_{T^*}, \gamma_{T^*})$ .

Alternatively, we impose a cap  $\bar{d} = d^*$  on the CEO's debt such that she chooses precisely an amount of debt  $d^*$ . The shareholder's optimal contract and the level of effort chosen by the CEO must jointly satisfy the incentive compatibility constraint (27) and the participation constraint (28).

We show first that  $p_{T^*}$  is suboptimal for the CEO in this case. By definition,  $\frac{\partial V(p_{T^*}, m)}{\partial p} = \frac{\partial V(p, d^*, m)}{\partial p}$ . Replace  $p_{T^*}$  in (27) for  $\bar{d} = d^*$ . Hence, comparing (13) and (27) it follows that  $\gamma_d = \gamma_{T^*}$ . Replace the later in (28). By definition,  $V(p_{T^*}, d^*, m) = V(p_{T^*}, m)$ . Given (A21) it follows that, for any  $T^* > 0$ ,  $\gamma_{T^*} V(p_{T^*}, d^*, m) - c(p_{T^*}) - A < 0$ . In other words, the contract  $(0, \gamma_{T^*})$  and the debt cap  $\bar{d} = d^*$  induce the CEO's effort  $p_{T^*}$  and the socially efficient level of debt  $d^*$  but undercompensate the CEO.

Finally, we show that, to obtain the socially efficient level of debt, the optimal contract with a cap on debt must include a larger variable compensation to induce a larger amount of effort. Let us solve for  $c'(p)$  in (27) and replace it in (28). We can then express the participation constraint as a function of the CEO's effort as follows:

$$c'(p) \left( \frac{\partial V(p, d^*, m)}{\partial p} \right)^{-1} V(p, d^*, m) - c(p) - A = 0 \quad (\text{A22})$$

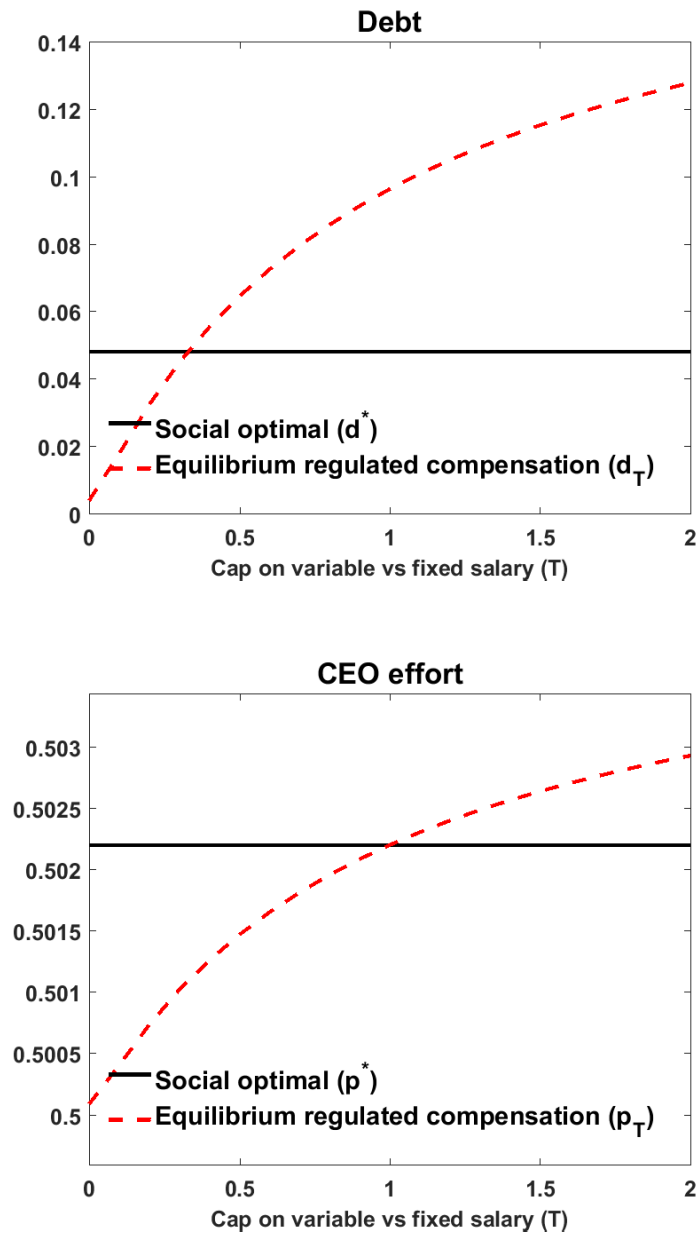
Taking the partial derivative of the left-hand side with respect to  $p$  yields

$$c''(p) \left( \frac{\partial V(p, d^*, m)}{\partial p} \right)^{-1} V(p, d^*, m) > 0.$$

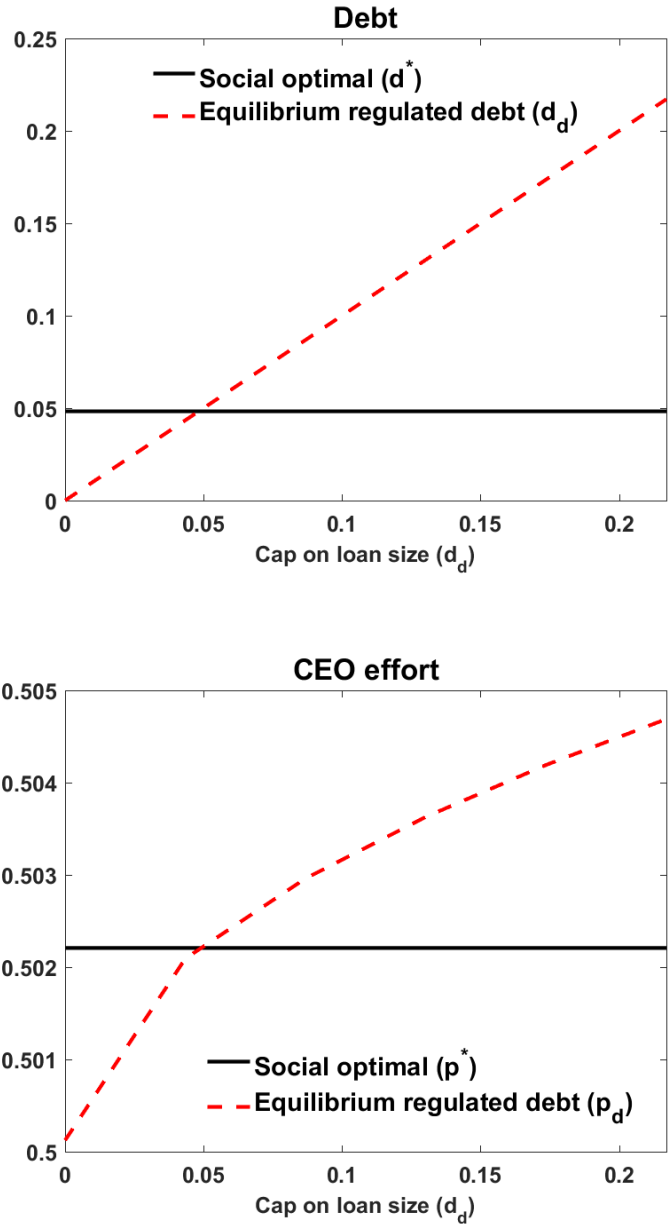
Hence, to satisfy (A22) the optimal effort  $p_{d^*} > p_{T^*}$ . Finally, given the incentive compatibility constraint (27), inducing higher effort implies that  $\gamma_{d^*} > \gamma_{T^*}$ .



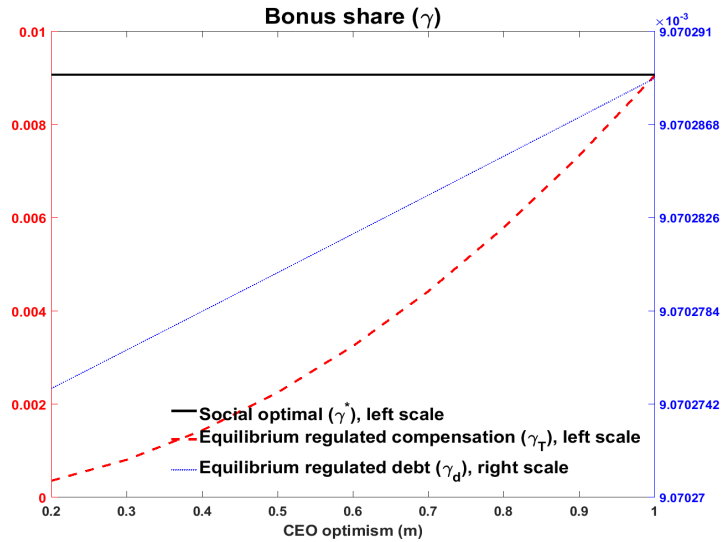
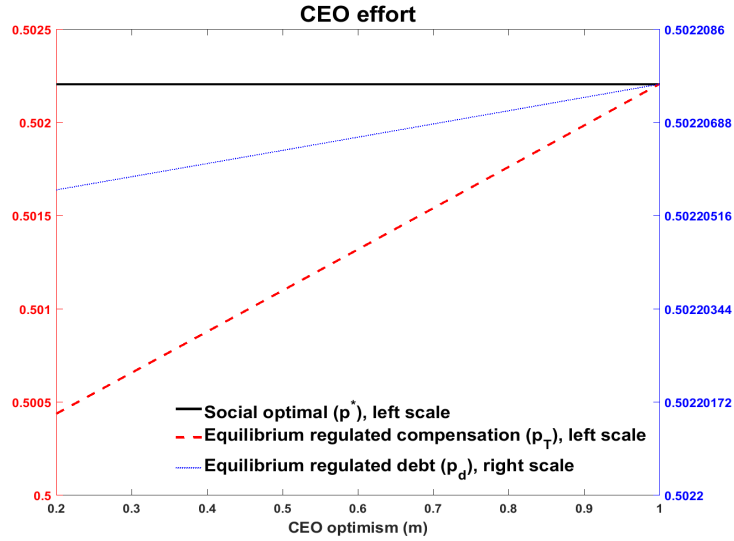
# Figures



**Figure 1. Regulating Executive Compensation.** This figure compares the market equilibrium with an optimistic CEO ( $m = 0.5$ ) with the socially optimal case (rational CEO, i.e.,  $m = 1$ ) for different caps on variable versus fixed compensation on the horizontal axis. The top panel shows debt while the bottom panel shows CEO's effort on the vertical axis.



**Figure 2. Regulating Leverage.** This figure compares the market equilibrium with an optimistic CEO ( $m = 0.5$ ) with the socially optimal case (rational CEO, i.e.,  $m = 1$ ) for different caps on leverage on the horizontal axis. The top panel shows debt while the bottom panel shows CEO's effort on the vertical axis.



**Figure 3. Comparing the Regulatory Tools.** This figure compares the market equilibrium under regulatory caps on compensation and leverage that achieve the optimal level of leverage ( $\bar{d} = d_T = d^*$ ) for different levels of CEO optimism,  $m$ , on the horizontal axis. The top panel plots the CEO's effort while the bottom panel plots, on the vertical axis, the variable  $\gamma$  controlling the variable payments. The figure includes the socially optimal case (rational CEO, i.e.,  $m = 1$ ).