Lending Standards and Macroeconomic Dynamics

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Abstract

This paper proposes a tractable way to incorporate lending standards ("credit qualification thresholds") into macro models of financial frictions. Banks can reject borrowers whose risk is above an endogenous threshold at which no lending rate sufficiently compensates banks for the borrower's default risk. Firms denied credit cut employment and labor reallocates mostly towards safer producers. Lending standards propagate bank capital shortfalls through labor misallocation causing deeper and more persistent real effects. The paper also shows that lending spreads are insufficient indicators of credit supply disruptions. That is, for the same increase in credit spreads, output falls faster when denial rates are increasing. Finally, with endogenous lending standards, first-moment bank capital shocks look as second-moment shocks.

Keywords: Bank Capital; Bank Losses; Extensive Margin; Lending Standards; Labor Reallocation; Misallocation.

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1 Introduction

The 2007-08 crisis has triggered a new literature that incorporates an intermediation sector into general equilibrium models. The goal is to study how shocks to financial intermediaries ("banks" for short) affect the real economy (see for example the surveys in Gertler 2012 or Guerrieri et al. 2018). In this paper I incorporate lending standards into such literature. I allow banks to adjust not only along credit spreads, but also through rejection rates. That is, banks optimally set lending cut-off rules ("the lending standards") to reject the riskiest borrowers. Laufer and Paciorek (2016) and Jimenez et al. (2018) document this behavior of banks. For example, in mortgage markets banks deny applications whose risk is above a threshold inferred from the credit scores of the borrowers. Rodano, Serrano-Velarde and Tarantino (2017) find that most Italian banks adjust their lending standards through higher rejection rates.

Incorporating lending standards into a macro model of financial frictions is interesting for three reasons: 1) it generates a new transmission channel of bank capital shortfalls through labor reallocation that is supported by recent applied work. For example, Bentolila et al. (2018) and Puri, Rocholl and Steffen (2011) document significant increases in loan rejection rates following banks' equity losses. De Jonghe et al. (2016) and Kok and Schepens (2013) show that banks facing negative funding shocks reallocate credit towards low-risk firms. Berton et al. (2018), Chodorow-Reich (2014), Duygan-Bump, Levkov and Montoriol-Garriga (2015) and Popov and Rocholl (2016) document that contractions of credit supply lead to labor reallocation. Huber (2018) shows that banking shocks lower productivity. 2) The paper shows that credit spreads may be insufficient indicators of credit supply disruptions when banks are also changing their denial rates. That is, for the same increase in credit spreads, output is falling more when denial rates are increasing. Consistent with this result, Lown and Morgan (2006) and Bassett et al. (2014) show that changes in lending standards affect output even when controlling for credit spreads. 3) With endogenous lending standards, first-moment bank capital shocks increase cross-sectional output variances like in the second-moment shocks popular since Christiano, Motto and Rostagno (2014).

In the model, there are entrepreneurs (firms), households and banks. Entrepreneurs use their equity and bank loans to hire labor and produce output. Banks fund loans using their own equity and borrowings from households. Entrepreneurs are subject to idiosyncratic productivity shocks and costly-state verification frictions, as in BGG, that generate equilibrium default and endogenous credit spreads over banks' costs of funds. However, departing from the literature, I assume that entrepreneurs are heterogenous in the variance ("risk" for short) and the mean of their idiosyncratic shocks. Banks have a screening technology to perfectly infer if the risk of a borrower is above or below a lending standards threshold. Banks deny credit applications from those borrowers whose risk is above the threshold because, with debt contracts, banks' expected revenue is decreasing in borrower's risk, which increases the probability of default. The threshold is an endogenous rationing limit which depends on banks' costs of funds and expectations of borrower's revenue. Changes in the threshold induce changes in the extensive margin of credit. This is consistent with Laufer and Paciorek (2016), who show that lenders' minimum credit score lending rules fluctuate over time getting tighter with the financial crisis.

In the model, a bank capital shortfall leads to higher borrowing costs for banks because there is a second costly-state verification friction between banks and their lenders. This is consistent with the evidence discussed in Bindseil and Laeven (2017) that interbank markets (banks' lenders in the model) ex-ante do not observe how credit losses are distributed across individual banks because bank lending is opaque. Thus, when banks lose equity their borrowing costs increase as banks' lenders require compensation since a greater amount of leverage makes bank default more likely.

To illustrate the propagation channel generated by lending standards, first I assume that households' labor is in fixed supply. In this environment, an exogenous shock to banks' equity has no effect on output when banks cannot deny credit applications. That is, in a model with only intensive credit margin and inelastic labor supply, output is constant. This is result is due to lower wages that counteract the higher credit spreads caused by the bank capital shortfalls. Guerrieri et al. (2018) discuss how this result applies to all the existing macro models of banks, even if they allow for physical capital, because with financial shocks the immediate fall in output has to ride through a contraction in hours worked. Calibration choices regarding the Frisch elasticity of labor supply become key for the immediate output reduction. For this reason, it is common in the literature to trigger a banking crisis with a real shock, like a reduction of capital quality as in Gertler and Karadi (2011).

When banks can tighten their lending standards, financial crises cause output falls, even with inelastic labor supply. Entrepreneurs denied credit reduce employment. Labor reallocates to the safest producers which are still endowed with credit, and to those producers that are so risky that could not get credit even before the banking crisis but now can expand because wages fall. In a model with lending standards, output decreases for two novel reasons: (1) the safer producers become excessively large and because of decreasing returns to scale productivity falls; (2) Risk and productivity are positively related, then tighter lending standards reallocate labor towards safer but low-productivity producers. Therefore, increase in employment by the safest and riskiest entrepreneurs cannot compensate for the output losses from those denied credit because of the tightened lending standards.

Methodologically the paper contributes to the growing literature that analyzes DSGE models with an intermediation sector. For example, some recent publications are Ajello (2016), Andreasen, Ferman and Zabczyk (2013), Angeloni and Faia (2013), Boissay, Collard and Smets (2016), Bocola (2016), Collard et al. (2017), De Fiore, Teles and Tristani (2011), Ferrante (2018), Gertler, Kiyotaki and Queralto (2012), Gertler and Karadi (2011), Gertler and Kiyotaki (2015), Iacoviello (2015) or Ravn (2016) among others. To the best of my knowledge, no paper in this literature allows for lending standards as a non-price mechanism. Thus, no paper allows lenders to deny credit to the riskier borrowers causing the labor misallocation that is at the core of this paper.

Kudlyak and Sanchez (2017) find that after the third quarter of 2008 large firms contracted much more than small firms. They hypothesize that it might be the case that small versus large is not a good approximation of the debt constrained versus unconstrained dimension for firms. This paper shows that risky versus safe may be a more adequate approximation.

The paper complements the misallocation literature that argues that financial frictions can cause output losses through input reallocation. For example, Arellano, Bai and Kehoe (2016), Buera and Moll (2015), Buera, Jaef and Shin (2015), Khan, Senga and Thomas (2014) and Siemer (2014) among others. This literature takes financial shocks as exogenous, usually assuming shocks to the collateral constraints without modelling the financial sector. This paper elaborates more on the banks' side. Agents are constrained or not depending on the dynamics of the endogenous lending standards, which depend on borrower and banks' variables as the cost of funds.

This paper shows a way to capture misallocation that can be solved with the linearization techniques used in policy DSGE models as Christiano, Eichenbaum, and Evans (2005) or Smets and Wouters (2007). In this regard the paper complements Debortoli and Gali (2017), who discuss that many insights from fully heterogeneous agents models can be captured by DSGE models with fewer degrees of heterogeneity.

The paper is organized as follows. Section 2 presents the model. Section 3 calibrates it. Section 4 contains the results. Section 5 concludes.

2 Model

It is a general equilibrium model with households, entrepreneurs and banks. Households consume goods, sell their labor to the entrepreneurs and deposit with the banks, which then lend to the entrepreneurs. The entrepreneurs produce goods and finance their labor costs with their equity and, if approved by the banks, with bank loans. The model is real and consumption is the numeraire.

2.1 Households

There is a continuum of homogeneous households who maximize expected utility over consumption C_t , hours worked H_t^H , and deposits D_t

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(C_t, H_t^H), \tag{1}$$

The budget constraint of the representative household is

$$C_t + D_t^H = W_t H_t^H + R_t D_{t-1}^H + \Pi_t^E + \Pi_t^B,$$
(2)

where W_t is the wage per labor unit and Π_t^E and Π_t^B are dividends paid by the entrepreneurs and banks respectively. R_t is the return on deposits, which is endogenous and state-contingent as described below.¹

2.2 Entrepreneurs

Every period there is a continuum of heterogeneous entrepreneurs with mass one. Each entrepreneur i produces output at t according to the function

$$y\left(\omega_t^i, h_{t-1}^i\right) = z_t \omega_t^i \left(h_{t-1}^i\right)^{\alpha},\tag{3}$$

where h_{t-1}^i is the number of labor units hired last period, ω_t^i is the idiosyncratic productivity shock, and z_t is the total-factor-productivity (TFP) shock common across entrepreneurs. The parameter $\alpha < 1$ generates decreasing returns to scale.

¹It is not important for the mechanism in the paper whether deposits are state-contingent or not.

The TFP shock follows an AR(1) process

$$\log\left(z_{t}\right) = \rho_{z}\log\left(z_{t-1}\right) + \epsilon_{z,t},\tag{4}$$

where $\epsilon_{z,t} \sim N(0, \sigma_z)$.

The idiosyncratic productivity shock ω_t^i is i.i.d. across periods, like in Bernanke, Gertler and Gilchrist (1999, BGG). However, a novelty of this paper is that entrepreneurs are heterogeneous in the distribution function of the ω_t^i shocks. That is, ω_t^i comes from a log-normal distribution where the standard deviation σ_t^i and the mean μ_t^i of the back-transformed (logged) Normal distribution are specific to each entrepreneur,

$$\omega_t^i \sim \ln N\left(\mu_t^i, \sigma_t^i\right). \tag{5}$$

That is, once the entrepreneur hires labor she receives an i.i.d. random draw σ_t^i . Thus, entrepreneurs are heterogeneous in their idiosyncratic risk and return. The entrepreneur's μ_t^i is a function of σ_t^i ,

$$\mu_t^i = \kappa \ln\left(\sigma_t^i\right) - \frac{\left(\sigma_t^i\right)^2}{2}.$$
(6)

where κ is a parameter that controls the risk-return tradeoff. That is, the function (6) implies a tractable expression for the expected productivity of the entrepreneur that only depends on entrepreneur's σ_t^i ,

$$\mathbb{E}\left(\omega_t^i\right) = \int_0^\infty \omega_t^i dF^i\left(\omega_t^i; \sigma_t^i\right) = \exp\left(\mu_t^i + \frac{\left(\sigma_t^i\right)^2}{2}\right) = \left(\sigma_t^i\right)^\kappa,\tag{7}$$

where $F^i(\omega_t^i; \sigma_t^i)$ denotes the cumulative distribution function associated to (5). If $\kappa > 0$, $\mathbb{E}_t(\omega^i)$ is increasing in risk σ_t^i , that is, the riskiest entrepreneurs are the most productive. If $\kappa = 0$, then $\mathbb{E}_t(\omega^i) = 1$, that is, expected productivity is the same for all entrepreneurs although some are riskier than others.

The random draws σ_t^i come from a log-normal distribution whose parameters μ_{σ} and σ_{σ} are the same for all the entrepreneurs and constant over time,

$$\ln\left(\sigma_t^i\right) \sim N\left(\mu_{\sigma}, \sigma_{\sigma}\right). \tag{8}$$

 $H(\sigma_t^i)$ will denote the cumulative distribution function associated to (8).

It is important to remark that the entrepreneurs of this model converge to the entrepreneurs

of BGG when $H(\sigma_t^i)$ is a degenerated distribution. The model converges into Christiano, Motto and Rostagno (2014) when the cross-sectional dispersion of idiosyncratic productivity is the same across entrepreneurs although time-varying, that is, $\sigma_t^i = \sigma_t \forall i$.

2.3 The demand for credit

Entrepreneurs have equity N_t^E and can obtain a loan of size l_t at an endogenous borrowing rate R_t^L . Entrepreneurs optimally choose their demand for credit to maximize expected profits before knowing their idiosyncratic productivity. That is, entrepreneurs solve

$$\max_{h_t, l_t, R_t^L} \mathbb{E}_t \left[\int_0^\infty \left[\int_{\hat{\omega}_{t+1}}^\infty \left[\omega_{t+1}^i z_{t+1} h_t^\alpha - R_t^L l_t \right] dF^i \left(\omega_{t+1}^i; \sigma_t^i \right) \right] dH \left(\sigma_t^i \right) \right], \tag{9}$$

subject to (10), (11), and (12) defined below.

Equation (9) defines entrepreneurs' expected profits. It integrates over the idiosyncratic volatility σ_t^i because the entrepreneur ignores her type. The inner integral in (9) captures that entrepreneurs have limited liability. That is, if the entrepreneur defaults ($\omega_t^i < \hat{\omega}_t$) the lender seizes the entrepreneur's assets and the debt disappears. Entrepreneurs not defaulting earn the difference between their revenue minus the debt repayments.

Equation (10) is the entrepreneurs' balance sheet. Entrepreneurs use their funds (equity and loans) to hire workers and pay the adjustment costs associated with changing their employed labor force. These adjustment costs are controlled by the parameter $\psi > 0$ and create stickiness in loan size.

$$N_t^E + l_t = W_t h_t + \psi \left(l_t - l_{t-1} \right)^2, \tag{10}$$

Equation (11) defines the endogenous default threshold, $\hat{\omega}_t$, such that, once the aggregate and idiosyncratic shocks arrive, those entrepreneurs unable to repay $(\omega_t^i < \hat{\omega}_t)$ default,

$$R_{t-1}^{L}l_{t-1} = \hat{\omega}_t z_t h_{t-1}^{\alpha}, \tag{11}$$

Equation (12) is the standard zero-profit condition for lenders that determines lending rates in BGG models and that entrepreneurs think that determine their borrowing costs, R_t^L ,

$$\mathbb{E}_{t} \left[\int_{0}^{\infty} \left[\begin{array}{c} \int_{\hat{\omega}_{t+1}}^{\infty} R_{t}^{L} l_{t} dF^{i} \left(\omega_{t+1}^{i}; \sigma_{t}^{i} \right) + \\ + (1 - \mu_{E}) \int_{0}^{\hat{\omega}_{t+1}} \omega_{t+1}^{i} z_{t+1} \left(h_{t}^{U} \right)^{\alpha} dF^{i} \left(\omega_{t+1}^{i}; \sigma_{t}^{i} \right) \right] dH \left(\sigma_{t}^{i} \right) \right] = R_{t}^{b} l_{t}.$$
(12)

The left hand side of (12) is lenders' expected revenue from the loan, that is, revenue from those entrepreneurs able to repay and from seizing the value of the defaulted entrepreneur net of monitoring costs μ_E . The right hand side is the lender's cost of funds R_t^b , which are determined below. Equations (9) and (12) assume that when entrepreneurs apply for credit they ignore banks' lending standards defined below. This is a reasonable assumption given that in reality banks do not publish their lending standards.

2.4 Lending standards

Banks screen entrepreneurs and observe whether a borrower with risk σ_t^i is above or below an endogenous lending standard threshold $\bar{\sigma}_t$. Laufer and Paciorek (2016) document that this is how banks operate in mortgage markets. Lenders set time-varying minimum thresholds for acceptable credit scores and reject all credit applications whose risk is larger than the threshold. Similarly, I assume that for a loan application of size l_t , the bank screens and denies applications that are too risky in the sense that $\sigma_t^i > \bar{\sigma}_t$. That is, banks follow the lending rule

$$l_t^i \left(\sigma_t^i \right) = \left\{ \begin{array}{c} 0 \text{ if } \sigma_t^i > \bar{\sigma}_t \\ \\ \\ l_t \text{ if } \sigma_t^i \le \bar{\sigma}_t \end{array} \right\}$$
(13)

for an endogenous lending standard threshold $\bar{\sigma}_t$ determined below.

The lending rule (13) assumes that the bank can do a risk assessment and ensure that no borrower above a certain risk-level is financed. However, the bank cannot condition the loan amount on the risk of the borrower. This assumption simplifies the numerical solution of the model because all borrowers below $\bar{\sigma}_t$ receive the same credit conditions.

Banks determine the optimal lending standard threshold $\bar{\sigma}_t$ as the riskiest borrower at which they can break-even. That is, banks' expected revenue from a borrower with loan size l_t , lending rate $R_{t,i}^L$ and idiosyncratic type σ_t^i is

$$\Omega\left(R_{t,i}^{L},\sigma_{t}^{i},l_{t},h_{t}\right) = \mathbb{E}_{t} \begin{bmatrix} \int_{\hat{\omega}_{t+1}^{i}}^{\infty} R_{t,i}^{L} l_{t} dF^{i}\left(\omega_{t+1}^{i};\sigma_{t}^{i}\right) + \\ + \int_{0}^{\hat{\omega}_{t+1}^{i}} (1-\mu_{E}) \omega_{t+1}^{i} z_{t+1} h_{t}^{\alpha} dF^{i}\left(\omega_{t+1}^{i};\sigma_{t}^{i}\right) \end{bmatrix},$$
(14)

where $\hat{\omega}_{t+1}^i$ is the function of $R_{t,i}^L$ defined by equation (11). The first integral of (14) is the revenue for the bank when the borrower is expected to repay. The second integral is the revenue when the borrower defaults.

 \bar{R}_t^L is the lending rate that maximizes banks' expected revenue for a borrower with loan size l_t and idiosyncratic type σ_t^i ,

$$\bar{R}_{t}^{L} = \underset{R_{t}^{L}}{\arg\max} \ \Omega\left(R_{t,i}^{L}, \sigma_{t}^{i}, l_{t}, h_{t}\right), \tag{15}$$

$$s.t.$$
 (11). (16)

Figure 1 shows that the maximized revenue $\Omega\left(\bar{R}_{t}^{L}, \sigma_{t}^{i}, l_{t}, h_{t}\right)$ is usually decreasing in σ_{t}^{i} . Higher σ_{t}^{i} increases the probability of borrower's default $\left(\int_{0}^{\hat{\omega}_{t+1}^{i}} dF^{i}\left(\omega_{t+1}^{i}; \sigma_{t}^{i}\right)\right)$ and decreases lenders' expected revenue.

Insert Figure 1 here

The optimal lending standard threshold $\bar{\sigma}_t$ is the intersection in Figure 1 of the maximum revenue that the bank can obtain with the banks' cost of funds R_t^b . That is, $\bar{\sigma}_t$ is the borrower at which her maximum revenue covers the banks' costs of funds.

Formally, given the function $\bar{R}_t^L(\sigma_t^i)$ defined in (15), $\bar{\sigma}_t$ solves

$$\mathbb{E}_{t} \begin{bmatrix} \int_{\bar{\omega}_{t+1}}^{\infty} \bar{R}_{t}^{L} l_{t} dF^{i} \left(\omega_{t+1}^{i}; \bar{\sigma}_{t}\right) + \\ + \int_{0}^{\bar{\omega}_{t+1}} z_{t+1} \left(1 - \mu_{E}\right) \omega_{t+1}^{i} h_{t}^{\alpha} dF^{i} \left(\omega_{t+1}^{i}; \bar{\sigma}_{t}\right) \end{bmatrix} = R_{t}^{b} l_{t}, \qquad (17)$$

where the default threshold $\bar{\omega}_{t+1}$ is the function of \bar{R}_t^L specified in equation (11). For borrowers with risk above $\bar{\sigma}_t$ there is no rate at which the bank can cover its cost of funds. Equations (15) and (17) determine the lending standards $\bar{\sigma}_t$ and lending rate \bar{R}_t^L maximizing banks' revenue.

It is important to stress that \bar{R}_t^L is not the rate at which the financed entrepreneurs, $\sigma_t^i \in (0, \bar{\sigma}_t]$, borrow. Banks' lending rate is the rate \hat{R}_t^L at which the competitive banks expect to break even given a level of lending standards. That is, \hat{R}_t^L is determined by lenders' zero-profit condition given the lending standards:

$$\mathbb{E}_{t}\left[\int_{0}^{\bar{\sigma}_{t}}\left[\begin{array}{c}\int_{\check{\omega}_{t+1}}^{\infty}\hat{R}_{t}^{L}l_{t}dF^{i}\left(\omega_{t+1}^{i};\sigma_{t}^{i}\right)+\\\\+\left(1-\mu_{E}\right)\int_{0}^{\check{\omega}_{t+1}}\omega_{t+1}^{i}z_{t+1}h_{t}^{\alpha}dF^{i}\left(\omega_{t+1}^{i};\sigma_{t}^{i}\right)\end{array}\right]dH\left(\sigma_{t}^{i}\right)\right]=\int_{0}^{\bar{\sigma}_{t}}R_{t}^{b}l_{t}dH\left(\sigma_{t}^{i}\right).$$
 (18)

Equation (18) coincides with (12) when the lending standard threshold is infinite.

The effective lending rate \hat{R}_t^L determines the effective default threshold $\check{\omega}_{t+1}$,

$$\hat{R}_t^L l_t = \check{\omega}_{t+1} z_{t+1} h_t^{\alpha}. \tag{19}$$

To illustrate how the lending standards alter the link between banks' lending rates and cost of funds, Figure 2 plots the banks' lending rate \hat{R}_t^L implied by equation (18) as a function of the banks' cost of funds R_t^b for different lending standards thresholds $\bar{\sigma}_t$.

Insert Figure 2 here

For any $\bar{\sigma}_t$, the lending rate \hat{R}_t^L is increasing in banks' cost of funds R_t^b since competitive banks need to pass their higher borrowing costs to the entrepreneurs. However, tight lending standards (lower $\bar{\sigma}_t$) lower interest rates. That is, there are less defaults when banks filter out the high-risk borrowers. Thus, competitive banks can offer lower lending rates when their lending standards are tight.

2.5 Banks

There is a continuum of banks with mass one. The total amount of credit B_t^E extended by the banks is the aggregate of the loans given to the qualified borrowers:

$$B_t^E = \int_0^{\bar{\sigma}_t} l_t dH\left(\sigma_t^i\right). \tag{20}$$

Banks finance B_t^E with their equity N_t^B , and with borrowings B_t^B ,

$$B_t^E = N_t^B + B_t^B. (21)$$

The realized revenue for the banks is

$$\Psi_{t+1}\left(\hat{R}_{t}^{L}, l_{t}, \bar{\sigma}_{t}\right) = \int_{0}^{\bar{\sigma}_{t}} \begin{bmatrix} \int_{\check{\omega}_{t+1}}^{\infty} \hat{R}_{t}^{L} l_{t} dF^{i}\left(\omega_{t+1}^{i}; \sigma_{t}^{i}\right) + \\ + (1-\mu_{E}) \int_{0}^{\check{\omega}_{t+1}} \omega_{t+1}^{i} z_{t+1} h_{t}^{\alpha} dF^{i}\left(\omega_{t+1}^{i}; \sigma_{t}^{i}\right) \end{bmatrix} dH\left(\sigma_{t}^{i}\right).$$
(22)

Banks are subject to idiosyncratic i.i.d. shocks u with mean one, $\mathbb{E}[u] = 1$. The u shocks are realized in the period when the banks need to return their borrowings. That is, $u\Psi_{t+1}$ is the effective revenue for a bank with shock u. The u shocks capture that some banks hold high quality loans while others hold low quality loans and may default. Thus, the lenders of the banks (the depositors) are exposed to bank default risk and will price this risk with an endogenous spread over the lenders' return on their funds. This is consistent with the evidence discussed in Bindseil and Laeven (2017) that interbank markets (the lenders of the banks in the model) ex-ante do not observe how credit losses are distributed across individual banks because bank lending is opaque. The u shocks are Pareto distributed with cumulative density function G(u).²

Banks borrow at rate R_t^d . Those banks with shocks u_t below the default threshold \hat{u}_t default, where \hat{u}_t is defined by

$$\widehat{u}_t \Psi_t = R^d_{t-1} B^B_{t-1}.$$
(23)

The return on bank deposits is the revenue from the banks repaying R_t^d plus the revenue (net of bankruptcy cost μ_B) from those banks that default.

$$R_t B_{t-1}^B = \int_{\widehat{u}_t}^{\infty} R_{t-1}^d B_{t-1}^B dG(u) + (1-\mu_B) \int_0^{\widehat{u}_t} u \Psi_t dG(u).$$
(24)

I assume that the owners of banks' equity require a return equal to the banks' depositors. This is reasonable since in the model depositors are equally exposed to banks' default risk. Thus, the cost of funds for the banks is the banks' borrowing rate,

$$R_t^b = R_t^d. (25)$$

2.6 Market clearing

Output per entrepreneur is

$$Y_t^i\left(\sigma_{t-1}^i\right) = \left\{ \begin{array}{l} \int_0^\infty \omega_t^i z_t \left(\frac{N_{t-1}^E}{W_{t-1}}\right)^\alpha dF^i\left(\omega_t^i;\sigma_{t-1}^i\right) \text{ if } \sigma_{t-1}^i > \bar{\sigma}_{t-1} \\ \\ \int_0^\infty \omega_t^i z_t h_{t-1}^\alpha dF^i\left(\omega_t^i;\sigma_{t-1}^i\right) \text{ if } \sigma_{t-1}^i \le \bar{\sigma}_{t-1} \end{array} \right\}.$$

$$(26)$$

The previous definition assumes parameter configurations such that rejected entrepreneurs use all their equity to hire workers. That is, entrepreneurs are credit constrained and would borrow if they could do so.

Aggregate output is the sum of the production of the entrepreneurs receiving credit and

²The Pareto distribution improves the quantitative properties of the model relative to other distributions as the Log-normal distribution. The parameters of the Pareto are the scale parameter u_{\min} and the shape parameter α_B which controls the dispersion of the shocks.

those rejected credit:

$$Y_{t} = \int_{0}^{\bar{\sigma}_{t-1}} \int_{0}^{\infty} \omega_{t}^{i} z_{t} h_{t-1}^{\alpha} dF^{i} \left(\omega_{t}^{i}; \sigma_{t-1}^{i}\right) dH \left(\sigma_{t-1}^{i}\right) + \int_{\bar{\sigma}_{t-1}}^{\infty} \int_{0}^{\infty} \omega_{t}^{i} z_{t} \left(\frac{N_{t-1}^{E}}{W_{t-1}}\right)^{\alpha} dF^{i} \left(\omega_{t}^{i}; \sigma_{t-1}^{i}\right) dH \left(\sigma_{t-1}^{i}\right).$$
(27)

Labor market clearing requires that the sum of the labor demand by the entrepreneurs receiving credit, and by those rejected credit, equals the endogenous labor supply of the households plus the exogenous labor supply of entrepreneurs and banks:

$$h_t \int_0^{\bar{\sigma}_t} dH\left(\sigma_t^i\right) + \frac{N_t^E}{W_t} \int_{\bar{\sigma}_t}^\infty dH\left(\sigma_t^i\right) = H_t^H + H^E + H^B.$$
(28)

The labor supply of entrepreneurs and banks is exogenous and constant over time. Assuming labor supply for banks and entrepreneurs is common in models of financial frictions to ensure inflows into the equity of entrepreneurs and banks.

Credit markets clear when household's deposits equal banks' borrowings:

$$D_t^H = B_t^B. (29)$$

2.7 Equity laws of motion

The aggregate profits of the entrepreneurs are the profits of the financed entrepreneurs, plus the profits of the credit-rationed entrepreneurs:

$$V_{t} = \left[\int_{0}^{\bar{\sigma}_{t-1}} \left[\begin{array}{c} \int_{\tilde{\omega}_{t}}^{\infty} \omega_{t}^{i} z_{t} \left(h_{t-1}\right)^{\alpha} dF^{i} \left(\omega_{t}^{i}; \sigma_{t-1}^{i}\right) + \\ - \int_{\tilde{\omega}_{t}}^{\infty} \hat{R}_{t-1}^{L} l_{t-1} dF^{i} \left(\omega_{t}^{i}; \sigma_{t-1}^{i}\right) \end{array} \right] dH \left(\sigma_{t-1}^{i}\right) \right] +$$

$$(30)$$

$$+ \int_{\bar{\sigma}_{t-1}}^{\infty} \int_{0}^{\infty} \omega_{t}^{i} z_{t} \left(\frac{N_{t-1}^{E}}{W_{t-1}}\right)^{\alpha} dF^{i} \left(\omega_{t}^{i}; \sigma_{t-1}^{i}\right) dH \left(\sigma_{t-1}^{i}\right).$$

$$(31)$$

Entrepreneurs and banks pay their profits minus the retained earnings as dividends to the households:

$$\Pi_t^E = (1 - \gamma_E) V_t, \tag{32}$$

$$\Pi_{t}^{B} = (1 - \gamma_{B}) \int_{\widehat{u}_{t}}^{\infty} \left[u \Psi_{t} \left(\hat{R}_{t-1}^{L}, l_{t-1}, \bar{\sigma}_{t-1} \right) - R_{t-1}^{d} B_{t-1}^{B} \right] dG(u).$$
(33)

Entrepreneurs' equity evolves as retained earnings plus labor income:

$$N_t^E = \gamma_E V_t + W_{t-1} H^E. \tag{34}$$

Banks' aggregate equity N_t^B is the sum of past retained profits and bankers' labor income, plus an exogenous shock T_t that triggers the crises that I study below,

$$N_t^B = \gamma_B \int_{\widehat{u}_t}^{\infty} \left[u \Psi_t - R_{t-1}^d B_{t-1}^B \right] dG(u) + W_{t-1} H^B + T_t, \tag{35}$$

$$T_t = \rho_T T_{t-1} + \epsilon_{T,t},\tag{36}$$

where $\epsilon_{T,t} \sim N(0, \sigma_T)$. In steady state there are no exogenous shocks for the banks.

3 Calibration

I assume GHH preferences with parameters v, θ and γ ,

$$U(C_t, H_t^H) = \frac{\left[C_t - v\frac{\left(H_t^H\right)^{\theta}}{\theta}\right]^{1-\gamma} - 1}{1-\gamma}.$$
(37)

To parameterize the model I split the parameters in two groups. First, exogenous parameters that are standard in the literature. Second, endogenous parameters to match some reasonable targets. Table 1 reports the parameters and Table 2 contains the targets. One period in the model is one quarter.

The exogenous parameters are: (i) the share of labor in output $\alpha = 0.6$; (ii) the discount factor $\beta = \frac{1}{1.01}$ that generates a 1% quarterly risk-free lending rate in steady state. I refer to this rate as the prime rate; (iii) a risk aversion parameter $\gamma = 2$; (iv) an elasticity of labor supply of $\frac{1}{\theta-1} = 1.67$; (v) credit adjustment cost $\psi = 7$; (vi) persistence of the bank equity shock $\rho_T = 0.5$; (vii) persistence of TFP shock $\rho_z = 0.9.3$

The endogenous parameters $(\kappa, \alpha_B, \mu_E, \mu_B, H_E, H_B, \gamma_E, \gamma_B, \mu_\sigma, \sigma_\sigma)$ allow the model to match ³I normalize the labor supply from households H^H to 1 to pin down v.

the following annualized targets at steady state:⁴ (i) A spread between banks' borrowing costs and the prime rate of around 2%. In the U.S., the spread between interbank loans and government debt (the TED spread) fluctuates between 1% and 3%. (ii) A spread between banks' lending rate and the prime rate of around 7%. This value seems a reasonable credit spread for small firms. It is a middle-ground between the APRs on credit cards and on banks' loans. The Small Business Administration reports spreads on small business loans of 2.25-6.5% over the prime rate in 2017. In addition, many small businesses borrow with credit cards with APR above 10%. (iii) A loan approval rate of 77.34%, in the range of 76% - 80% reported by the Small Business Credit Survey of 2016-2017. (iv) The default rate of entrepreneurs is around 5%, like the estimate of Fernandez and Gulan (2015), and within the 4.96% to 5.37%range reported by De Fiore and Uhlig (2011) for the E.U. and U.S.; (v) The default rate of banks is 3.76%, which is consistent with the 2% to 6% range reported by IMF (2007); (vi) The weighted leverage ratio of credit constrained and unconstrained entrepreneurs is 4.48, close to the 4.27 estimated by Fernandez and Gulan (2015); (vii) Banks' equity to assets ratio is 12.4%, which is within the recent range for most OECD banks (Kaul and Goodman 2016); (viii) The ratio of loans-to-entrepreneurs-to-GDP is 41%. It is around the average bank-credit-to-privatenonfinancial-sector as percentage of GDP in the U.S. over 1990 - 2017 as by BIS; (ix) The ratio of total-loans-of-banks-to-GDP is 35.82%. The ratio matches the median level (35%) across 185 countries of the average bank-deposits-to-GDP ratio over 1990 - 2015. (x) The labor-costto-GDP ratio is 53%, which is in line with the average labor compensation to GDP (59%) of non-financial corporate sector in the U.S. over 2010 - 2017.

Insert Tables 1 and 2 here

4 Results

The model presented above is basically the benchmark Bernanke, Gertler and Gilchrist (1999, BGG) with two key differences: 1) borrowers are heterogeneous in idiosyncratic risk; and 2) lenders can implement lending thresholds such that the riskiest borrowers are denied credit. The model is designed in such way that can be solved with the standard linearization techniques used to solve DSGE models. In this section I show that those two differences generate interesting mechanisms that are new in the literature.

⁴Since I assume that the mean of banks' idiosyncratic i.i.d. shocks u is 1, the scale parameter of the Pareto distribution u_{\min} is automatically determined.

The formulas for targets can be found in Appendix Definitions.

First, I show how the model with lending standards differs in steady state from a model without them ($\bar{\sigma}_t = \infty, \forall t$). The main insight is that the lending standards make output per entrepreneur to follow a bimodal distribution. This will generate a new transmission channel from financial shocks to output that I analyze next. Moreover, in the model with lending standards, first moment shocks to bank equity look as second moment shocks. Finally, I discuss how the model with lending standards behaves after the TFP shocks common in RBC models.

4.1 Steady state properties

Figures 3 and 4 are based on the steady state of the model. Figure 3 plots output per entrepreneur Y_t^i defined in (26) as a function of entrepreneur's volatility σ_t^i . Figure 4 plots the distribution of output per entrepreneur in an economy with lending standards ($\bar{\sigma}_t < \infty, \forall t$) and in one without them ($\bar{\sigma}_t = \infty, \forall t$).

Figure 3 shows two insights: 1) Above and below the lending standard threshold, output is increasing and concave in entrepreneur's volatility σ_t^i . These patterns come from the shape of $\mathbb{E}_t(\omega^i)$ defined in (7) for $\kappa > 0$. That is, the riskier producers are the more productive. 2) There is a sharp decrease in output per entrepreneur after the lending standard cutoff. Banks deny credit to the borrowers that are too risky for them to expect to break even. Hence, the most productive entrepreneurs, because they are too risky, become smaller as they only fund their production with their own equity.

Insert Figure 3 here

As a consequence, Figure 4 shows a bimodal distribution in the model with lending standards ($\bar{\sigma}_t < \infty$, $\forall t$) relative to a unimodal distribution in the model without lending standards ($\bar{\sigma}_t = \infty$, $\forall t$). The right-tail of the output distribution in the model in which every entrepreneur gets credit ($\bar{\sigma}_t = \infty$, $\forall t$) shifts to the left in the economy with lending standards.

Insert Figure 4 here

Next, I will show that these differences in output per entrepreneur caused by the lending standards generate novel transmission channels after bank capital shortfalls.

4.2 Banking crises

To illustrate the novel channel proposed by this paper, Figures 5 and 6 plot the financial and real effects of a bank capital shortfall when households' labor supply is inelastic. The figures compare models with and without lending standards.⁵ The size of the shock is such that banks lose 20% of equity at the shock impact, then bank capital evolves endogenously following (35). I focus first on the inelastic labor supply because for this case, in a model with no lending standards ($\bar{\sigma}_t = \infty, \forall t$), financial shocks cause no effect in output. Thus, all effects on output in the model with lending standards are due to the new transmission channels. Then Table 3 reports results from the model with endogenous labor supply.

Insert Figures 5 and 6 here

The upper left panel of Figure 5 shows the exogenous bank equity shock T_t hitting both models. In both the models with and without lending standards a bank capital shortfall increases banks' borrowing costs as banks' lenders price the higher risk of bank default. In both models, banks react by charging higher lending rates and loan size falls. Moreover, in the model with lending standards, banks also increase their rejection rates since their costs of funds are higher.⁶

Figure 6 shows the key differences across models. Output only falls in the model with endogenous lending standards. The model without lending standards is basically a standard BGG model. All borrowers receive credit and hire the same number of workers. Employment per entrepreneur does not change in a banking crisis. Moreover, if labor is in fixed supply output has to stay constant because labor supply did not change. Thus, bank capital shortfalls generate no output fall because, to clear labor markets, wages decrease and counteract the higher borrowing costs that entrepreneurs face. Guerrieri et al. (2018) discuss that this result also holds for all models with financial shocks in which credit is used to buy capital and capital is pre-determined (i.e., fixed at the shock impact).

Figure 6 shows that the model with lending standards has a new transmission channel because employment per entrepreneur does change. The bank capital shortfall triggers tighter lending standards which reduce credit to the riskiest but most productive entrepreneurs. This generates the reallocation of labor shown in Figures 3 and 4 and lower output.

⁵The calibration ensures that bank leverage is the same whether the model has or not lending standards. To do so, I alter the banks' labor supply parameter H^B in the model without lending standards.

⁶The presence of adjustment costs to loan size is key to avoid that this fall in loan size is so large than lending standards relax.

To better illustrate that endogenous lending standards propagate bank capital shortfalls through labor misallocation, Figure 7 decomposes the changes in output across type of entrepreneurs. We can see that those entrepreneurs never financed and those always financed expand their employment as they benefit from lower wages. However, entrepreneurs rejected credit dramatically cut their employment. That is, when banks can screen and change their approval rates, the banking crisis is associated with a labor reallocation from the mid-risk entrepreneurs now denied credit towards the safest borrowers who still qualify for credit, and towards the riskiest entrepreneurs, which were already excluded from credit markets before the shock and are able now to expand when wages are lower. Two different mechanisms lower output: 1) Because of decreasing returns to scale, as the labor size of unconstrained entrepreneurs become too large, the labor productivity decreases (as the upper-right panel of Figure 7 shows); 2) Since risk and productivity are positively related ($\kappa > 0$) as banks move away from risk then the share of workers employed with the more productive entrepreneurs falls. As a consequence, even with inelastic labor supply, bank capital shortfalls lower output.

Insert Figure 7 here

Table 3 illustrates that the previous mechanism is also present in models with endogenous labor supply. When labor supply can drop as wages fall, then both models with and without lending standards generate output effects from bank capital shortfalls. However, the productivity channel associated with the labor reallocation induced by tighter standards generate larger output falls.

Insert Table 3 here

The model with lending standards generate two other interesting results in a banking crisis:

First, Figure 5 shows that tighter standards imply that the lending rates to entrepreneurs, and loan size to those financed, change by less in the model with lending standards. This is because with tighter lending standards banks reduce their exposure to risky borrowers. Thus, the likelihood of default of the financed borrowers is lower. Banks can extend credit in better conditions when their standards are tighter. These results imply that lending spreads may be insufficient indicators of credit supply disruptions when banks are also changing their denial rates. That is, for the same increase in credit spreads, output falls more when denial rates are increasing. This result may explain why the applied literature on lending standards finds predictive power in this variable, even when controlling for credit spreads.

Second, Figure 8 plots the cross-sectional variance of output after the bank capital shortfall

studied in Figures 5, 6 and 7. Labor reallocation induces higher cross-sectional variance in output. Thus, the model with lending standards and first-moment equity shocks generates second-moment effects as if instead we input a cross sectional shock à la Christiano, Motto and Rostagno (2014).

Insert Figure 8 here

4.3 TFP shocks

So far we studied bank capital shocks. Figures 9 and 10 analyze the interaction between TFP shocks and lending standards. Figure 9 plots how steady state bank revenue, $\Psi_{t+1}\left(\hat{R}_t^L, l_t, \bar{\sigma}_t\right)$, changes when TFP falls for different levels of lending standards with the same amount of credit. Tight standards cushion the impact of negative TFP shocks. The reason is that the economy with tighter standards see lower increases in defaults as banks are less exposed to the riskier borrowers who are more likely to default as TFP falls.

Insert Figure 9 here

Figure 10 displays the dynamics after a standard mean reverting TFP shock with endogenous labor supply. After the shock, entrepreneurs' profitability is lower and their risk of default higher. Banks tighten standards and trigger the labor reallocation channel discussed before. If we define measured TFP as

$$\frac{Y_t}{\left(H_{t-1}^H + H^E + H^B\right)^{\alpha}}$$

then Figure 10 shows that TFP falls because of the exogenous shock but, in the model with lending standards, also because of the labor reallocation channel. In the model without lending standards TFP dynamics are exogenous and follow the shock z_t . Thus, Figure 10 shows that the model with lending standards have more mechanisms that can amplify the effects of shocks.

Insert Figure 10 here

5 Conclusions

This paper proposed a tractable way to incorporate lending standards into DSGE models of financial frictions. Tighter lending standards is a non-price mechanism that may generate misallocation between safe and risky borrowers. Facing higher borrowing costs, banks reject the riskier borrowers at which no lending rate sufficiently compensates banks for the borrower's default risk.

Lending standards generate labor misallocation. Employment flows towards the safest producers that become too large and aggregate productivity falls. The effect is reinforced when the riskier borrowers are the more productive entrepreneurs. Thus, the model with lending standards have new transmission channels relative to the standard DSGE with financial frictions.

The paper shows that lending spreads are insufficient indicators of credit supply disruptions when banks are also changing their denial rates. That is, for the same increase in credit spreads, increasing denial rates lower output more. This result may explain why the applied literature on lending standards finds predictive power in this variable, even when controlling for credit spreads.

With endogenous lending standards, first-moment bank capital shocks look as secondmoment shocks. That is, the cross-sectional distribution of output increases as in Christiano, Motto and Rostagno (2014), who directly input cross sectional shocks.

An avenue for future research is to incorporate the channel of this paper into a DSGE model which can be estimated and could evaluate the importance of price versus non-price credit mechanisms. The mechanism of lending standards studied in this paper could also be important when evaluating the transmission channels of monetary policy.

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Tables

Description	Parameter	Value
Exogenously Determ	ined	
Labor share in production	α	0.6
Discount factor	β	$\frac{1}{1.01}$
Risk aversion	γ	2
Labor supply elasticity	$\frac{1}{\theta-1}$	1.67
Credit adjustment cost	ψ	7
Persistence of bank equity shock	$ ho_T$	0.5
Persistence of TFP shock	$ ho_z$	0.9
Endogenously Deter	mined	
Risk-return parameter	κ	0.05
Shape Pareto distribution of banks	α_B	8
Entrepreneurs' default cost	μ_E	0.9
Banks' default cost	μ_B	0.5
Entrepreneurs' labor supply	H_E	0.0025
Banks' labor supply	H_B	0.0025
Entrepreneurs retaining rate	γ_E	0.2
Banks retaining rate	γ_B	0.96
Mean of $\ln(\sigma_i)$	μ_{σ}	-1.80
Standard deviation of $\ln(\sigma_i)$	σ_{σ}	1.9

Table 2: Steady State Moments and Targets (annualized)				
	Target	Model		
Banks' borrowing spread	1% - 3%	1.98%		
Banks' lending spread	6.75% - 9.25%	7.04%		
Entrepreneurs' approval rate	76-80%	77.46%		
Default rate of entrepreneurs	5.1%	5.03%		
Default rate of banks	2%-6%	3.76%		
Weighted leverage ratio of all entrepreneurs	4.263	4.48		
Equity-to-assets of banks	7%-13%	12.4%		
Total loan of entrepreneurs to GDP ratio	49%	41%		
Total loan of banks to GDP ratio	35%	35.82%		
Labor cost to GDP ratio	59%	53%		

Cumulative losses as % of steady-state level. One year after shock					
	Endogenous labor supply		Exogenous labor supply		
	No Lending Standards (LS)	With LS	No LS	With LS	
Output	5.22%	8.57%	0%	1.64%	
Employment	12.41%	14.78%	-	-	
Measured TFP	0%	1.83%	0%	1.64%	

 Table 3: Comparing Transmission Channels After Bank Capital Shortfall

Note: "No LS" means the model with no lending standards ($\bar{\sigma}_t = \infty, \forall t$). The initial shock wipes out 20% of steady state bank equity.

Figures

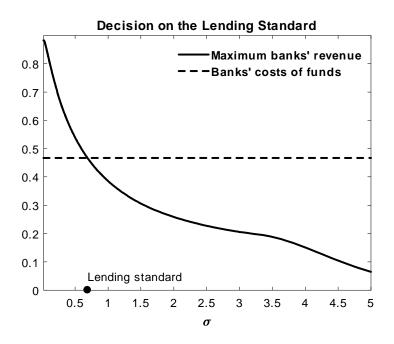


Figure 1. Borrower's risk and bank's revenue. This figure plots the maximum bank's revenue (Ω) as a function of entrepreneurs' idiosyncratic productivity (σ) . That is, the solid line is $\Omega\left(\bar{R}_t^L, \sigma_t^i, l_t, h_t\right)$ as defined by (15) as a function of entrepreneurs' idiosyncratic productivity. The dashed line plots a level of banks' cost of funds. The optimal lending standard threshold is the borrower at which her maximum revenue covers the banks' costs of funds.

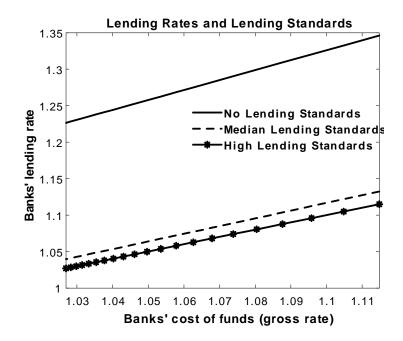


Figure 2. Banks' lending rate and lending standards. Each line plots banks' lending rate \hat{R}_t^L as a function of banks' cost of funds R_t^b for different levels of lending standards $\bar{\sigma}_t$. Banks' lending rate \hat{R}_t^L is defined by the banks' participation condition (18).

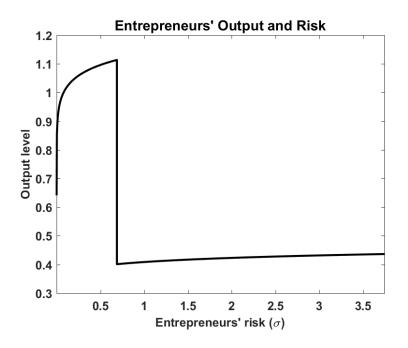


Figure 3. Output as a function of entrepreneur's volatility. This figure plots entrepreneur's output Y_t^i defined in equation (26) as a function of entrepreneur's volatility σ_t^i .

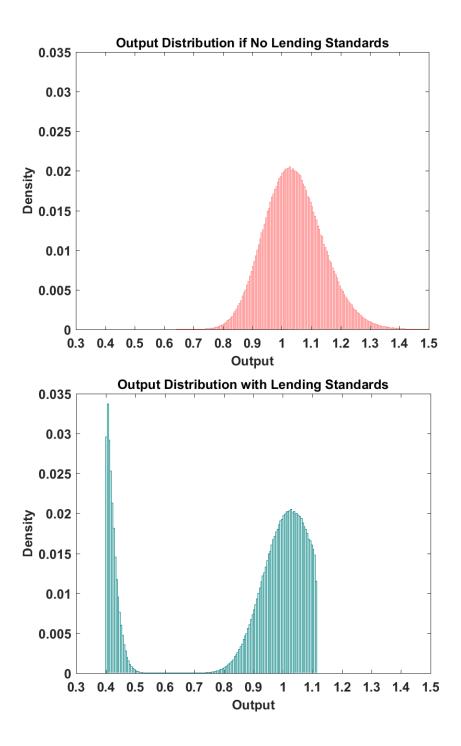


Figure 4. Distribution of output per entrepreneur in the models with and without lending standards. The top panel plots the distribution of output per entrepreneur in the model with no lending standards ($\bar{\sigma}_t = \infty, \forall t$). The bottom panel plots the distribution of output per entrepreneur in the model with lending standards ($\bar{\sigma}_t < \infty, \forall t$).

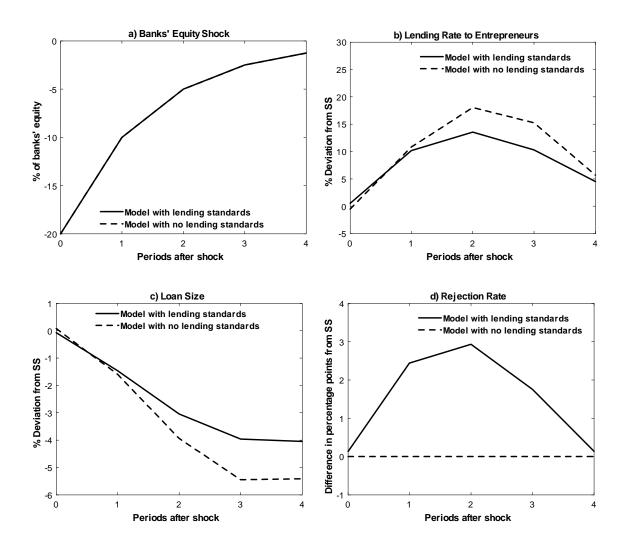


Figure 5. Financial effects of bank capital shortfall with exogenous labor supply. This figure plots impulse responses following a negative shock to bank capital. In this figure the model assumes exogenous labor supply. The dashed line is a model with no lending standards ($\bar{\sigma}_t = \infty, \forall t$). The solid line is the benchmark model with lending standards ($\bar{\sigma}_t < \infty, \forall t$).

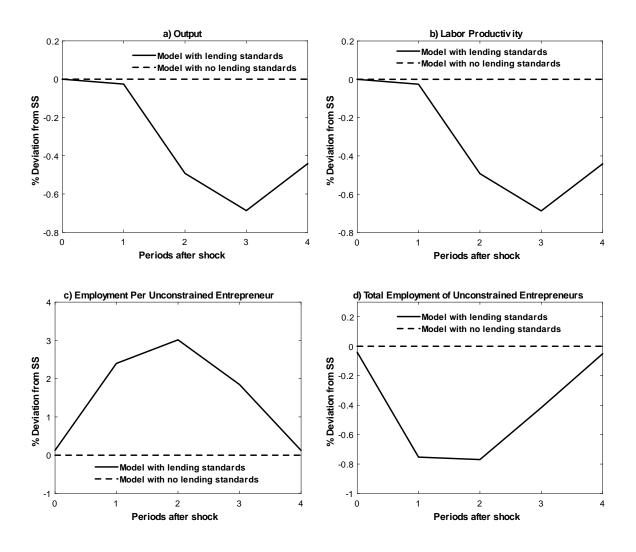


Figure 6. Real effects of bank capital shortfall with exogenous labor supply. This figure plots impulse responses following the negative shock to bank capital reported in Figure 5. In this figure the model assumes exogenous labor supply. The dashed line is a model with no lending standards $(\bar{\sigma}_t = \infty, \forall t)$. The solid line is the benchmark model with lending standards $(\bar{\sigma}_t < \infty, \forall t)$.

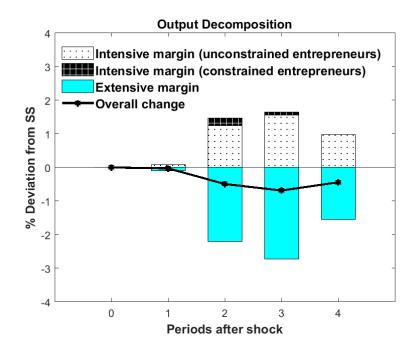


Figure 7. Output decomposition following a bank capital shortfall in model with lending standards. This figure plots the change in output of different groups of entrepreneurs following the negative shock to bank capital reported in Figure 5. It is the model with lending standards ($\bar{\sigma}_t < \infty$, $\forall t$) and exogenous labor supply. The white bars show the output change due to the entrepreneurs who always get credit (intensive margin unconstrained entrepreneurs). The black bars shows the output change due to the entrepreneurs who are always denied credit (intensive margin constrained entrepreneurs). The blue-shaded bars show the output change due to the entrepreneurs financed before the shock but not after it (extensive margin).

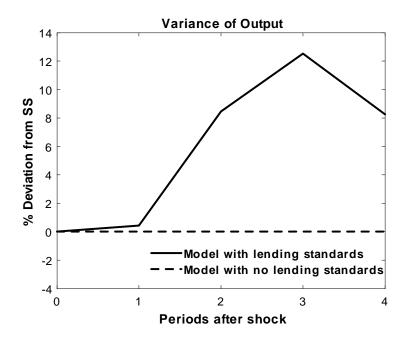


Figure 8. Output variance following a bank capital shortfall in model with lending standards. This figure plots the cross-sectional variance of output after the bank capital shortfall studied in Figures 5, 6 and 7. The dashed line is a model with no lending standards $(\bar{\sigma}_t = \infty, \forall t)$. The solid line is the benchmark model with lending standards $(\bar{\sigma}_t < \infty, \forall t)$.

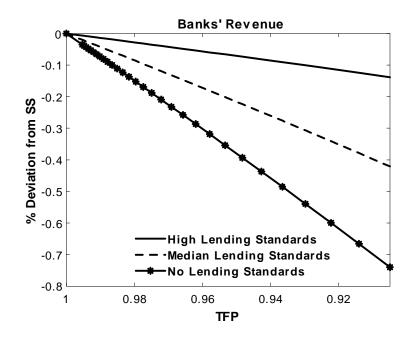


Figure 9. Banks' revenue, TFP and lending standards. This figure plots banks' aggregate revenue Ψ defined in equation (22) as a function of TFP (z) for different levels of lending standards. To facilitate the comparison, the total amount of credit B^E is the same in the three lines.

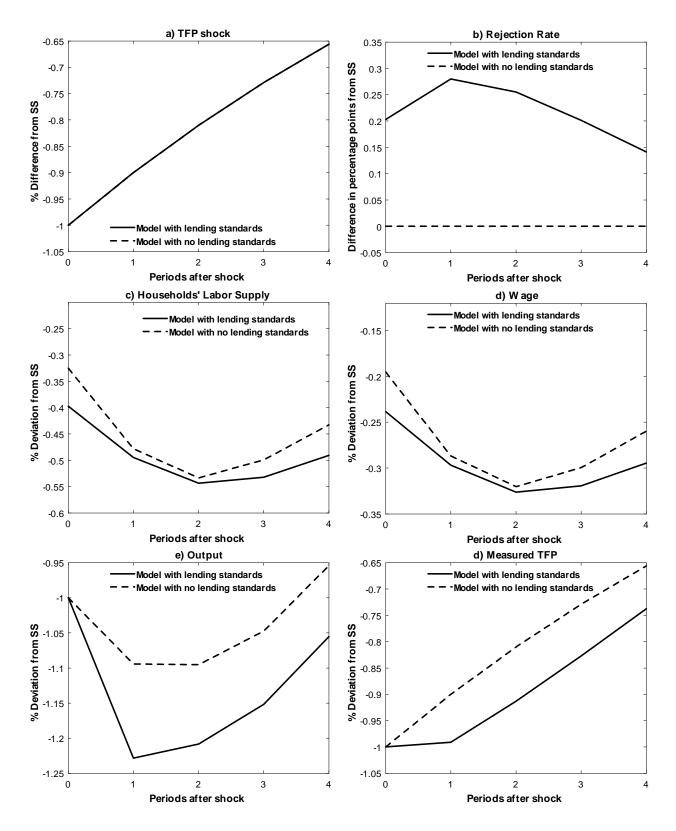


Figure 10. Effects of TFP shock when labor supply is endogenous. This figure plots impulse responses following a negative shock to TFP. In this figure the model assumes endogenous labor supply. The dashed line is a model with no lending standards ($\bar{\sigma}_t = \infty, \forall t$).

Online Appendix: Not-For-Publication

A First-order Conditions

A.1 Households

The households maximize (1) s.t. (2). The Lagrangian function is:

$$\pounds^{H} = \mathbb{E}_{t} \left[\sum_{t=0}^{\infty} \beta^{t} u(C_{t}, H_{t}^{H}) + \lambda_{t}^{H} \left(W_{t} H_{t}^{H} + R_{t} D_{t-1}^{H} + \Pi_{t}^{E} + \Pi_{t}^{B} - C_{t} - D_{t}^{H} \right) \right].$$
(A.1)

The first-order-conditions to B^H and H^H are:

$$u_1(C_t, H_t^H) = \beta \mathbb{E}_t \left[u_1(C_{t+1}, H_{t+1}^H) R_{t+1} \right],$$
(A.2)

$$W_t = -\frac{u_2(C_t, H_t^H)}{u_1(C_t, H_t^H)}.$$
(A.3)

Using the GHH utility function:

$$U(C_t, H_t^H) = \frac{\left[C_t - v\frac{\left(H_t^H\right)^{\theta}}{\theta}\right]^{1-\gamma} - 1}{1-\gamma}.$$
(A.4)

We can derive (A.3) as

$$W_t = v \left(H_t^H \right)^{\theta - 1}. \tag{A.5}$$

A.2 Financial Contract

We can express the default threshold (11) as

$$\hat{\omega}_{t+1} = \frac{R_t^L l_t}{z_{t+1} (h_t)^{\alpha}},\tag{A.6}$$

from which we can derive

$$\frac{\partial \hat{\omega}_{t+1}}{\partial R_t^L} = \frac{l_t}{z_{t+1} \left(h_t\right)^{\alpha}},\tag{A.7}$$

$$\frac{\partial \hat{\omega}_{t+1}}{\partial l_t} = \frac{R_t^L}{z_{t+1} \left(h_t\right)^{\alpha}},\tag{A.8}$$

-

$$\frac{\partial \hat{\omega}_{t+1}}{\partial h_t} = \frac{-\alpha R_t^L l_t}{z_{t+1} \left(h_t\right)^{\alpha+1}}.$$
(A.9)

The entrepreneurs maximize (9) s.t. (10), (11) and (12) from Section 2.3. Incorporating (11) into the problem and defining λ_t and μ_t as the Lagrange multipliers for (10) and (12), the Lagrangian function becomes

$$\begin{aligned} \pounds^{f} &= \mathbb{E}_{t} \left[\int_{0}^{\infty} \left[\int_{\hat{\omega}_{t+1}}^{\infty} \omega_{t+1}^{i} z_{t+1} \left(h_{t}\right)^{\alpha} dF^{i} \left(\omega_{t+1}^{i}; \sigma_{t}^{i}\right) + \right] dH \left(\sigma_{t}^{i}\right) \right] + \\ &+ \lambda_{t} \left[\mathbb{E}_{t} \left[\int_{0}^{\infty} \left[\int_{\hat{\omega}_{t+1}}^{\infty} R_{t}^{L} l_{t} dF^{i} \left(\omega_{t+1}^{i}; \sigma_{t}^{i}\right) + \right] dH \left(\sigma_{t}^{i}\right) \right] - R_{t}^{b} l_{t} \right] + \\ &+ \mu_{t} \left[N_{t}^{E} + l_{t} - W_{t} h_{t} - \psi \left(l_{t} - l_{t-1}\right)^{2} \right]. \end{aligned}$$
(A.10)

The first-order-conditions (FOC) are:

• FOC relative to h_t^U :

$$\mathbb{E}_{t} \left[\int_{0}^{\infty} \left[\int_{\hat{\omega}_{t+1}}^{\infty} \omega_{t+1}^{i} z_{t+1} \alpha \left(h_{t}\right)^{\alpha-1} dF^{i} \left(\omega_{t+1}^{i}; \sigma_{t}^{i}\right) \right] dH \left(\sigma_{t}^{i}\right) \right] + \lambda_{t} \left[\mathbb{E}_{t} \left[\int_{0}^{\infty} \left[\left(1-\mu_{E}\right) \int_{0}^{\hat{\omega}_{t+1}} \omega_{t+1}^{i} z_{t+1} \alpha \left(h_{t}\right)^{\alpha-1} dF^{i} \left(\omega_{t+1}^{i}; \sigma_{t}^{i}\right) + \right] dH \left(\sigma_{t}^{i}\right) \right] \right] + -\mu_{E} \frac{\partial \hat{\omega}_{t+1}}{\partial h_{t}} \hat{\omega}_{t+1} z_{t+1} \left(h_{t}\right)^{\alpha} f^{i} \left(\hat{\omega}_{t+1}; \sigma_{t}^{i}\right) \right] dH \left(\sigma_{t}^{i}\right) \right] \right] + -\mu_{t} W_{t} = 0.$$
(A.11)

Inserting (A.9) into (A.11), we have

$$\mathbb{E}_{t} \left[\int_{0}^{\infty} \left[\int_{\hat{\omega}_{t+1}}^{\infty} \omega_{t+1}^{i} z_{t+1} \alpha \left(h_{t}\right)^{\alpha-1} dF^{i} \left(\omega_{t+1}^{i}; \sigma_{t}^{i}\right) \right] dH \left(\sigma_{t}^{i}\right) \right] + \lambda_{t} \left[\mathbb{E}_{t} \left[\int_{0}^{\infty} \left[\left(1-\mu_{E}\right) \int_{0}^{\hat{\omega}_{t+1}} \omega_{t+1}^{i} z_{t+1} \alpha \left(h_{t}\right)^{\alpha-1} dF^{i} \left(\omega_{t+1}^{i}; \sigma_{t}^{i}\right) + \right] dH \left(\sigma_{t}^{i}\right) \right] \right] + \mu_{E} \frac{\alpha R_{t}^{L} l_{t}}{h_{t}} \hat{\omega}_{t+1} f^{i} \left(\hat{\omega}_{t+1}; \sigma_{t}^{i}\right) \right] dH \left(\sigma_{t}^{i}\right) \right] \right] + \mu_{t} W_{t} = 0.$$
(A.12)

• FOC relative to l_t :

$$\mathbb{E}_{t} \left[\int_{0}^{\infty} \left[\int_{\hat{\omega}_{t+1}}^{\infty} R_{t}^{L} dF^{i} \left(\omega_{t+1}^{i}; \sigma_{t}^{i} \right) \right] dH \left(\sigma_{t}^{i} \right) \right] \\
= \lambda_{t} \left[\mathbb{E}_{t} \left[\int_{0}^{\infty} \left[\int_{\hat{\omega}_{t+1}}^{\infty} R_{t}^{L} dF^{i} \left(\omega_{t+1}^{i}; \sigma_{t}^{i} \right) - \mu_{E} \frac{\partial \hat{\omega}_{t+1}}{\partial l_{t}} \hat{\omega}_{t+1} z_{t+1} \left(h_{t} \right)^{\alpha} f^{i} \left(\hat{\omega}_{t+1}; \sigma_{t}^{i} \right) \right] dH \left(\sigma_{t}^{i} \right) \right] - R_{t}^{b} \right] + \mu_{t} \left(1 - 2\psi \left(l_{t} - l_{t-1} \right) \right). \tag{A.13}$$

Inserting (A.8) into (A.13), we obtain

$$\mathbb{E}_{t} \left[\int_{0}^{\infty} \left[\int_{\hat{\omega}_{t+1}}^{\infty} R_{t}^{L} dF^{i} \left(\omega_{t+1}^{i}; \sigma_{t}^{i} \right) \right] dH \left(\sigma_{t}^{i} \right) \right] \\
= \lambda_{t} \left[\mathbb{E}_{t} \left[\int_{0}^{\infty} \left[\int_{\hat{\omega}_{t+1}}^{\infty} R_{t}^{L} dF^{i} \left(\omega_{t+1}^{i}; \sigma_{t}^{i} \right) - \mu_{E} R_{t}^{L} \hat{\omega}_{t+1} f^{i} \left(\hat{\omega}_{t+1}; \sigma_{t}^{i} \right) \right] dH \left(\sigma_{t}^{i} \right) \right] - R_{t}^{b} \right] + \mu_{t} \left(1 - 2\psi \left(l_{t} - l_{t-1} \right) \right). \tag{A.14}$$

• FOC relative to R_t^L :

$$\mathbb{E}_{t}\left[\int_{0}^{\infty}\left(1-F^{i}\left(\hat{\omega}_{t+1};\sigma_{t}^{i}\right)\right)dH\left(\sigma_{t}^{i}\right)\right] = \lambda_{t}\mathbb{E}_{t}\left[\int_{0}^{\infty}\left[1-F^{i}\left(\hat{\omega}_{t+1};\sigma_{t}^{i}\right)-\mu_{E}\hat{\omega}_{t+1}f^{i}\left(\hat{\omega}_{t+1};\sigma_{t}^{i}\right)\right]dH\left(\sigma_{t}^{i}\right)\right]$$
(A.15)

A.3 Lending Standards

To solve (15) the FOC is:

$$\Omega\left(R_{t,i}^{L},\sigma_{t}^{i},l_{t},h_{t}\right) = \mathbb{E}_{t}\left[\begin{array}{c}\int_{\hat{\omega}_{t+1}^{i}}^{\infty}R_{t,i}^{L}l_{t}dF^{i}\left(\omega_{t+1}^{i};\sigma_{t}^{i}\right) + \\ +\int_{0}^{\hat{\omega}_{t+1}^{i}}\left(1-\mu_{E}\right)\omega_{t+1}^{i}z_{t+1}\left(h_{t}\right)^{\alpha}dF^{i}\left(\omega_{t+1}^{i};\sigma_{t}^{i}\right)\end{array}\right],$$
(A.16)

$$\frac{\partial\Omega\left(R_{t,i}^{L},\sigma_{t}^{i},l_{t},h_{t}\right)}{\partial R_{t,i}^{L}} = \mathbb{E}_{t} \left[\begin{array}{c} \int_{\hat{\omega}_{t+1}^{i}}^{\infty} l_{t}dF\left(\omega_{t+1}^{i};\sigma_{t}^{i}\right) - \frac{\partial\hat{\omega}_{t+1}^{i}}{\partial R_{t,i}^{L}}R_{t,i}^{L}l_{t}f^{i}\left(\hat{\omega}_{t+1}^{i};\sigma_{t}^{i}\right) + \\ + \frac{\partial\hat{\omega}_{t+1}^{i}}{\partial R_{t,i}^{L}}\left(1 - \mu_{E}\right)\hat{\omega}_{t+1}^{i}z_{t+1}\left(h_{t}\right)^{\alpha}f^{i}\left(\hat{\omega}_{t+1}^{i};\sigma_{t}^{i}\right) \end{array} \right] = 0. \quad (A.17)$$

Using (11) we can simplify to

$$\mathbb{E}_{t}\left[\int_{\hat{\omega}_{t+1}^{i}}^{\infty} l_{t} dF^{i}\left(\omega_{t+1}^{i};\sigma_{t}^{i}\right) - \mu_{E} \frac{\partial \hat{\omega}_{t+1}^{i}}{\partial R_{t,i}^{L}} \hat{\omega}_{t+1}^{i} z_{t+1}\left(h_{t}^{U}\right)^{\alpha} f^{i}\left(\hat{\omega}_{t+1}^{i};\sigma_{t}^{i}\right)\right] = 0, \qquad (A.18)$$

and inserting (A.7) into (A.18) we obtain

$$\mathbb{E}_t \left[\int_{\hat{\omega}_{t+1}^i}^{\infty} l_t dF^i \left(\omega_{t+1}^i; \sigma_t^i \right) - \mu_E l_t \hat{\omega}_{t+1}^i f^i \left(\hat{\omega}_{t+1}^i; \sigma_t^i \right) \right] = 0.$$
(A.19)

Cancelling l_t , we obtain that $\bar{R}_{t,i}^L$ is implicitly defined by

$$\mathbb{E}_{t}\left[\int_{\hat{\omega}_{t+1}^{i}}^{\infty} dF^{i}\left(\omega_{t+1}^{i};\sigma_{t}^{i}\right) - \mu_{E}\hat{\omega}_{t+1}^{i}f^{i}\left(\hat{\omega}_{t+1}^{i};\sigma_{t}^{i}\right)\right] = 0$$
(A.20)

Equation (A.20) shows that for each level of σ_t^i there is a default threshold $\hat{\omega}_{t+1}^i$ maximizing the banks' profits.

Online Appendix: Not-For-Publication

B Definitions

Banks' borrowing spread:

$$R^d - R. (B.1)$$

Banks' lending spread:

$$\hat{R}^L - R. \tag{B.2}$$

Entrepreneurs' approval rate:

$$\int_0^\sigma dH\left(\sigma^i\right).\tag{B.3}$$

Default rate of entrepreneurs:

$$\frac{\int_{0}^{\bar{\sigma}} \left[\int_{0}^{\bar{\omega}} dF^{i}\left(\omega^{i};\sigma^{i}\right) \right] dH\left(\sigma^{i}\right)}{H\left(\bar{\sigma}\right)}.$$
(B.4)

Default rate of banks:

 $G(\hat{u}).$ (B.5)

Weighted leverage of unconstrained and constrained entrepreneurs:

$$\left(\frac{l+N^E}{N^E}\right)H\left(\bar{\sigma}\right) + \left(\frac{N^E}{N^E}\right)\left(1 - H\left(\bar{\sigma}\right)\right). \tag{B.6}$$

Equity-to-assets of banks:

 $\frac{N^B}{B^E}.$ (B.7)

Total loan of entrepreneurs to GDP ratio:

$$\frac{B^E}{Y}.$$
 (B.8)

Total loan of banks to GDP ratio:

$$\frac{B^B}{Y}.$$
 (B.9)

Labor cost to GDP ratio:

$$\frac{\left(H^H + H^E + H^B\right)W}{Y}.$$
(B.10)

Rejection rate:

$$\int_{\bar{\sigma}_t}^{\infty} dH\left(\sigma_t^i\right). \tag{B.11}$$

Labor productivity:

$$\frac{Y_t}{H_{t-1}^H + H^E + H^B}.$$
 (B.12)

Total employment of unconstrained entrepreneurs:

$$H\left(\bar{\sigma}_{t}\right)h_{t}.\tag{B.13}$$

Measured TFP:

$$\frac{Y_t}{\left(H_{t-1}^H + H^E + H^B\right)^{\alpha}}.$$
(B.14)

Variance of output:

$$\int_{0}^{\bar{\sigma}_{t-1}} \left(\int_{0}^{\infty} \omega_{t}^{i} z_{t} \left(h_{t-1} \right)^{\alpha} dF^{i} \left(\omega_{t}^{i}; \sigma_{t-1}^{i} \right) - Y_{t} \right)^{2} dH \left(\sigma_{t-1}^{i} \right) + \\
+ \int_{\bar{\sigma}_{t-1}}^{\infty} \left(\int_{0}^{\infty} \omega_{t}^{i} z_{t} \left(\frac{N_{t-1}^{E}}{W_{t-1}} \right)^{\alpha} dF^{i} \left(\omega_{t}^{i}; \sigma_{t-1}^{i} \right) - Y_{t} \right)^{2} dH \left(\sigma_{t-1}^{i} \right). \tag{B.15}$$

At time t, the output decomposition $Y_t - Y_{SS}$:

- If $\bar{\sigma}_{t-1} < \bar{\sigma}_{ss}$, then
 - Intensive margin for unconstrained entrepreneurs:

$$\int_{0}^{\bar{\sigma}_{t-1}} \int_{0}^{\infty} \omega_t^i \left(z_t h_{t-1}^{\alpha} - z_{ss} h_{ss}^{\alpha} \right) dF^i \left(\omega_t^i; \sigma_{t-1}^i \right) dH \left(\sigma_{t-1}^i \right).$$
(B.16)

– Intensive margin for constrained entrepreneurs:

$$\int_{\bar{\sigma}_{ss}}^{\infty} \int_{0}^{\infty} \omega_{t}^{i} \left[z_{t} \left(\frac{N_{t-1}^{E}}{W_{t-1}} \right)^{\alpha} - z_{ss} \left(\frac{N_{ss}^{E}}{W_{ss}} \right)^{\alpha} \right] dF^{i} \left(\omega_{t}^{i}; \sigma_{t-1}^{i} \right) dH \left(\sigma_{t-1}^{i} \right).$$
(B.17)

- Extensive margin:

$$\left[z_t \left(\frac{N_{t-1}^E}{W_{t-1}}\right)^{\alpha} - z_{ss}h_{ss}^{\alpha}\right] \left(\begin{array}{c} \int_0^{\bar{\sigma}_{ss}} \int_0^\infty \omega_t^i dF^i\left(\omega_t^i;\sigma_{t-1}^i\right) dH\left(\sigma_{t-1}^i\right) + \\ -\int_0^{\bar{\sigma}_{t-1}} \int_0^\infty \omega_t^i dF^i\left(\omega_t^i;\sigma_{t-1}^i\right) dH\left(\sigma_{t-1}^i\right) \end{array}\right).$$
(B.18)

• If $\bar{\sigma}_{t-1} \geq \bar{\sigma}_{ss}$, then

– Intensive margin for unconstrained entrepreneurs:

$$\int_0^{\bar{\sigma}_{ss}} \int_0^\infty \omega_t^i \left(z_t h_{t-1}^\alpha - z_{ss} h_{ss}^\alpha \right) dF^i \left(\omega_t^i; \sigma_{t-1}^i \right) dH \left(\sigma_{t-1}^i \right).$$
(B.19)

– Intensive margin for constrained entrepreneurs:

$$\int_{\bar{\sigma}_{t-1}}^{\infty} \int_{0}^{\infty} \omega_{t}^{i} \left[z_{t} \left(\frac{N_{t-1}^{E}}{W_{t-1}} \right)^{\alpha} - z_{ss} \left(\frac{N_{ss}^{E}}{W_{ss}} \right)^{\alpha} \right] dF^{i} \left(\omega_{t}^{i}; \sigma_{t-1}^{i} \right) dH \left(\sigma_{t-1}^{i} \right).$$
(B.20)

- Extensive margin:

$$\left[z_t h_{t-1}^{\alpha} - z_{ss} \left(\frac{N_{ss}^E}{W_{ss}}\right)^{\alpha}\right] \left(\begin{array}{c} \int_0^{\bar{\sigma}_{t-1}} \int_0^\infty \omega_t^i dF^i\left(\omega_t^i; \sigma_{t-1}^i\right) dH\left(\sigma_{t-1}^i\right) + \\ - \int_0^{\bar{\sigma}_{ss}} \int_0^\infty \omega_t^i dF^i\left(\omega_t^i; \sigma_{t-1}^i\right) dH\left(\sigma_{t-1}^i\right) \end{array}\right).$$
(B.21)