Compensation Contracts and Fire Sales*

Pedro Gete† and Juan-Pedro Gómez‡

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Abstract

This paper analyzes the impact of remuneration practices on banks' risk-taking in a model with fire sales externalities. When these externalities are not internalized by a bank's shareholders and executives, borrowing and fire sales are higher than the socially optimal level. Our analysis shows that plain-vanilla equity fails to internalize fire sales externalities. Deferred equity and long-term bonuses unrelated to short-term profits can restore social efficiency. Bail-in bonds can achieve efficiency at a smaller cost since they allow for state-contingent payments. It is not the level but the composition of variable compensation that determines the inefficiency. Excessive regulation may lead to suboptimal levels of risk-taking. Government guarantees reinforce the fire sales externalities and the need for regulation.

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†Corresponding author. Georgetown University, GCER and IE Business School. 37th and O Sts., NW Washington, DC, USA 20057. Email: pg252@georgetown.edu
‡IE Business School. María de Molina 12. 28006 Madrid. Spain. Email: juanp.gomez@ie.edu

1 Introduction

Since the beginning of the "Great Recession," executive compensation at banks and other financial institutions has been the subject of an intense debate.[1] This paper analyzes the impact of remuneration practices on banks' risk-taking, captured by the level of short-term leverage, in a model with fire sales externalities. These externalities are at the center of the new macroprudential approach to regulation (Kashyap et al. 2011), but the literature has not yet studied executive compensation as a tool to address them. First, we show that when these externalities are not internalized by a bank's shareholders and executives, borrowing is higher than the socially optimal level. We then analyze four compensation structures proposed by the academic literature. Our objective is to study the ability of these structures to induce the socially optimal level of leverage and fire sales. We show that bailout guarantees reinforce the fire sales externalities.

Fire sales occur when financially distressed firms need to sell assets at prices below their value in a best-use scenario. Fire sales can be quite sizeable and lead to high discounts relative to face value. For instance, in March 2012, Spain's Banco Santander sold property-backed loans for EUR 750 million at a 62 percent discount to face value. In June of the same year, the UK's Lloyds sold property-backed loans for EUR 971 million after a discount of 52 percent.[2]

Fire sales *per se* need not be socially inefficient. They may simply represent a redistribution of wealth among agents. In that case we talk of an "unconstrained efficient" equilibrium. On the other hand, fire sales may generate negative fire sales externalities that affect social welfare. There are three main theories as to why fire sales are socially inefficient (see Shleifer and Vishny 2011 or Dávila 2014 for surveys). First, when markets are incomplete, fire sales are inefficient if they hammer asset sellers with higher marginal utility of consumption or investment. Examples of this mechanism can be found, for instance, in Geanakoplos and Polemarchakis (1986), Gromb and Vayanos (2002) or Lorenzoni, (2008). Second, fire sales are inefficient if they involve assets that serve as collateral. Some recent examples in the literature include Bianchi (2011), Benigno et al. (2012), Gersbach and Rochet (2012), Jeanne and Korinek (2010) or Stein (2012). Finally, fire sales are inefficient if the assets end up in the hands of buyers who mismanage them. Krugman (1998), Aguiar and Gopinath (2005) and Acharya et al. (2010) examine this hypothesis in the context of fire sales by domestic credit-constrained firms to foreign investors. In any of these cases, fire sales lead to welfare losses for society.

[1] There is, however, no consensus about the role of compensation in the crisis (see, for instance, Murphy 2012).

[2] "Banks set to shed EUR 20 billion property loans", Financial Times, September 23, 2012.

The literature refers to "constrained efficient" equilibrium, like in Bianchi (2011), as second-best scenarios where the fire sales inefficiency cannot be eliminated (for example, when markets are still incomplete, collateral constraints are present, or some buyers are more efficient than others), but the equilibrium has the "socially right" level of fire sales. That is, the borrower, when making the leverage decision, fully understands the effect of potential fire sales on equilibrium asset prices and chooses the right level of borrowing. Constrained inefficiency happens when borrowers fail to correctly internalize the link between their actions and asset prices; thus, they "overborrow" from a social perspective. For instance, overborrowing may happen when borrowers are atomistic and do not take into account the general equilibrium effect of asset sales on prices (Lorenzoni 2008, Stein 2012); or when borrowers differ from agents who face collateral constrains, and these borrowers only take into account their own utility when borrowing (Dávila 2014). We study if and under what conditions certain compensation structures lead to constrained efficient levels of leverage and fire sales.

We analyze the following compensation structures: 1) Equity; 2) Deferred equity; 3) Long-term bonuses; and 4) Bail-in bonds that are written down to equity if short-term profits yield long-term losses for the bank. Our choices are motivated by the policy discussion on regulating executive compensation. For example, in Europe, the Liikanen Report (published by a group of experts the E.U. Commission has appointed to reform the E.U. banking sector) proposed three measures: an absolute cap on overall compensation (possibly linked to paid-out dividends), a relative cap on the level of variable to fixed income, and claw-backs on deferred compensation. By contrast, in the U.S., the Squam Lake Report (French et al., 2010) explicitly recommends that governments "regulate the structure but not the level of executive compensation in financial firms." Namely, it advocates for the implementation of deferred contingent compensation schemes whereby financial institutions might be required to withhold part of the estimated dollar value of each executive's annual compensation (including cash, stock, and option grants), for several years. At the end of this period, employees would receive the fixed dollar amount of their deferred compensation if the firm has not declared bankruptcy or received government support.[3]

[3]Some of these recommendations have been reflected in recent regulatory changes. In the U.S., the Dodd-Frank Act requires regulatory agencies to prohibit compensation practices that encourage inappropriate risk-taking activities, albeit no final agreement among the agencies involved has been reached yet. In the Euro Area, the Capital Requirements Directive IV has established a cap on bonuses: they cannot exceed 100 per cent of salary (200 per cent if the company wins shareholder approval). In the U.K., the new Remuneration Code has extended the minimum deferral period of senior bank executives to seven years. At least 60 percent of awards of directors and other high-earners must be deferred. Even some banks have started to implement several of these measures on their own initiative. In 2013, UBS became the first big bank to give senior bankers bonuses in the form of "bail-in" bonds that can be wiped out if the bank's regulatory capital falls below 7 per cent, or in the case of a "non-viability" loss ("UBS leads way with bonuses shake-up", Financial Times, February 5, 2013).

In our model, a representative bank has long-term investments with a non-stochastic return. In the short term, the bank can leverage its performance by taking on short-term debt and investing in a project with random return. If the short-term investment turns out to be unprofitable, the bank cannot pay its creditors without selling its long-term assets. We can interpret this liquidity shortage, for example, as the need to comply with a minimum capital ratio requirement (see, for instance, Hanson et al. 2010 and the references therein). Additional debt is not available since, for instance, losses occur in a systemic crisis in which distressed banks have no access to debt markets. Moreover, we assume that the bank cannot raise equity due to the "debt overhang" problem identified by Myers (1977). Thus, the bank, in order to meet its debt payments, must sell the long-term asset (property-backed loans, for instance) at a price below the asset's net present value. Hence, we call these sales "fire sales."

First, we do not take a stand on the mechanism, among those previously discussed, that renders fire sales inefficient. We focus on the necessary condition, whatever the mechanism, for the equilibrium to be "constrained inefficient", that is, when borrowers do not correctly internalize the equilibrium price at which the asset will have to be sold. In that case, executives and shareholders under-estimate the cost of liquidity. They think that they will receive a price per asset sold higher than the price they will actually receive. Thus, the bank over-borrows relative to what is socially optimal. In the event of a crisis, fire sales are excessive and asset prices are over-depressed. Second, we use Krugman's (1998) mechanism to explicitly define the social welfare function that the regulator maximizes.

Regulating compensation can achieve superior outcomes because it alters the incentives of bank executives. First, plain-vanilla equity fails, on its own, to internalize fire sales externalities, as it fails to "penalize" short-term relative to long-term payoffs. Deferred equity and long-term bonuses unrelated to short-term profits can restore the efficiency loss induced by the externality. Long-term bonuses unrelated to short-term profits increase the opportunity cost of fire sales, thus, reducing fire sales. Deferred compensation works if agents value one dollar less in the future than in the present. If that is the case, then deferred compensation reduces the rewards from short-term debt and thus the incentives to leverage and sell at a discount in the case of a liquidity shock. Deferred compensation would be useless if it is placed in an interest-making account paying the same interest rate as the bank executives' discount rate. In fact, deferred compensation can be thought of as a tax on compensation, where the tax rate is the executives' discount rate. Bail-in bonds reduce incentives to short-term debt by paying equity in cases of bank distress, in which equity has no value. The advantage of bail-in bonds is that they are a "cheaper" way to provide incentives. They increase the opportunity cost of fire sales in periods with liquidity needs while avoiding any remuneration for executives in periods with no liquidity problems.

Our numerical exercises show that regulating the level of incentives can back-fire. Setting upper or lower bounds on the number of shares, deferred shares and/or the size of long-term bonuses may lead bank executives to an overcautious choice of debt and, ultimately, fire sales below the socially optimal level. Overall, our findings support the Squam Lake report's recommendation: regulating the level of executives compensation may be suboptimal.

Finally, we analyze how fire sales externalities interact with the existence of government guarantees on banks' losses. When banks enjoy either implicit, or explicit, government guarantees, a moral hazard problem arises: bankers have incentives to overinvest in risky assets. Government guarantees are usually considered one of the main arguments behind banks' excessive risk taking before the financial crisis. In section 7, we show that the addition of government guarantees, to the fire sales externalities of price-taking banks, reinforces the need for regulating executive compensation. With government guarantees, banks borrow even more, making fire sales yet more onerous to society. Thus, the regulation of executive compensation has to be more "aggressive" to curb the incentives for excessive leverage and costly fire sales.

Our paper makes contributions, and aims to connect, two broad literatures: the literature on executive compensation and the literature on macro-prudential regulation. On the one side, there is a large literature on regulating executive compensation in financial institutions (see, among others, John et al. 2000, Bebchuk and Spamann 2009, Bolton et al. 2011, Raviv and Sisli-Ciamarra 2013, or Thanassoulis 2014). This literature has mostly focused on risk-shifting problems and externalities from competition in labor markets. Our contribution to this literature is to study the implications of a fire sale externality for different types of compensation contracts. To the best of our knowledge, this macro-prudential angle to regulation has not been applied to executive compensation.

On the other side, numerous economists and policy-makers have highlighted fire sales as a justification for a macro-prudential approach to financial regulation (see, for example, Bernanke, 2008, Kashyap et al. 2011 or De Nicolò et al. 2012). Capital or liquidity requirements and Pigouvian taxes have been suggested as solutions to the failure of banks to internalize fire sales externalities (see Korinek and Mendoza 2013 for a recent survey). To the best of our knowledge, nobody has studied executive compensation as a policy tool to address fire sales externalities.[4] We contribute to this literature by analyzing whether the recommended compensation mechanisms suggested in the Liikanen and Squam Lake reports lead bank executives to internalize fire sale externalities; that is, to select constrained efficient levels of fire sales.

[4] In a related paper, Gete and Gomez (2014) study fire sales in a model with moral hazard, endogenous effort and long-term compensation.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the model's inefficiency. Section 4 compares the different compensation schemes. Section 5 provides an explicit analysis of the inefficiency. Section 6 shows the effects of introducing risk aversion. Section 7 shows the effects of adding bail-out guarantees. Section 8 concludes. The Appendix contains all the proofs.

2 Setup

There are two periods ($t = 1, 2$) and high and low states of nature in period one, $s = \{h, l\}$. The high state occurs with exogenous probability $p > 0$. If the state is high in period one, then we denote that the state is also high in period two. If the state in the first period is low, then the state in the second period is also low. We use superscripts to indicate states of the world, $s = \{h, l\}$, and subscripts to indicate time period, $t = \{1, 2\}$.

There is a continuum of small banks which we model as a single representative price-taker bank. At the start of period 1, the bank can borrow one-period debt, $d > 0$, at exogenously given gross interest $R > 1$. Debt returns gross interest a^h if the state of nature is high, and a^l if it is low, with $a^l < R < a^h$.

At the start of period 1, the bank has k long-term assets that, at the end of period 2, pay a non-stochastic gross return $b > 0$. At the end of period 1, the bank may sell f^s units of its long-term assets in state s, at price:

$$q(f^s) = R^{-1}b - vf^s, \tag{1}$$

where $v > 0$ is a parameter that captures the inefficiency associated with fire sales. For example, in the models of Krugman (1998), Aguiar and Gopinath (2005) or Acharya et al. (2010), v captures the inferior skills of the asset's buyers when in a financial crisis all the efficient buyers in a financial crisis (for example, other banks) are in distress and cannot buy. We elaborate on this parameter in Section 5. We define these sales as fire sales since, for positive sales $(f^s > 0)$, the price is always below the assets' net present value (NPV), that is, $q(f^s) < R^{-1}b$.

The bank's profits at the end of period 1 given state of nature s are:

$$\pi_1^s(d, f^s) = q(f^s)f^s + (a^s - R)\,d. \tag{2}$$

The bank's profits at the end of period 2, given state of nature s in period 1, are the returns of the unsold assets:

$$\pi_2^s(d, f^s) = b(k_0 - f^s). \tag{3}$$

To simplify, we assume that the bank's profits in period 1 cannot be reinvested in new assets, and that the bank cannot default. Thus, the bank's choices must always satisfy:

$$\pi_1^s(d, f^s) \geq 0, \tag{4}$$

$$\pi_2^s(d, f^s) \geq 0, \tag{5}$$

$$f^s \geq 0, \tag{6}$$

$$d \geq 0. \tag{7}$$

We restrict the parameter space to avoid corner solutions in which the bank does not use short-term debt:

$$\frac{1}{2} < \frac{(1-p)\left(R - a^l\right)}{p\left(a^h - R\right)} < 1. \tag{8}$$

That is, the expected rewards from debt are enough to encourage borrowings. We also assume the bank starts with enough long term assets. That is,

$$k_0 > \frac{R^{-1}b}{v}. \tag{9}$$

This assumption simplifies several proofs without loss of generality. $\frac{R^{-1}b}{v}$ is the level of fire sales at which $q(f)$ is zero. Fire sales will never exceed that threshold.

3 The Inefficiency

Fire sales externalities and moral hazard from government guarantees are usually discussed as the main externalities justifying macroprudential regulation (see, for example, Bernanke, 2008, Kashyap et al. 2011 or De Nicolò et al. 2012). First, we focus on fire sales externalities. Then, in section 7, we integrate the moral hazard from government guarantees.

As discussed in the introduction, there are several mechanisms that render these externalities socially inefficient. We focus first on the common denominator of all those mechanisms: banks are atomistic and, when borrowing, they do not take into account the general equilibrium effect of their asset sales on prices. We will compare a bank managed by risk-neutral shareholders who do not internalize equation (1) with a bank managed by a planner who, when deciding how much short term debt to borrow, understands that her asset sales would affect prices. That is, the planner internalizes equation (1). After that, in Section 5, we introduce the mechanism proposed by Krugman (1998). We show that the allocations selected by the planner are actually the Pareto Optimal allocations. We proceed in this way to emphasize which regulations on executive compensation deliver allocations that would be optimal for multiple types of models, and, specifically, for models in which the internalization of asset prices leads to constrained efficient allocations.

We use subindex p to denote the planner variables $(d_p, f_p^s, q_p^l, \Pi_p)$, and subindex s to denote the shareholders' variables $(d_s, f_s^s, q_s^l, \Pi_s)$, for upper index $s = \{h, l\}$. Shareholders choose optimal borrowing and state-contingent fire sales to maximize the sum of the bank's discounted expected profits:

$$\Pi = \underset{d,f^s}{max}\ \mathbb{E}\left(\pi_1 + R^{-1}\pi_2\right),$$

subject to (4)-(7). Shareholders take fire sales prices as given and independent of their actions. This is a consequence of banks being small compared to the whole banking sector. This is the key assumption in models of fire sales externalities. Lorenzoni (2008) discusses why private contracts fail to internalize their effect on equilibrium prices. Then, we compare the shareholder's outcomes to those of a planner who solves the same problem but takes into account the correct fire sales prices (1). The following proposition characterizes the results of the comparison:

Proposition 1 *Shareholders' choices are inefficient* $\left(\Pi_p - \Pi_s > 0 \text{ and } \frac{\partial(\Pi_p - \Pi_s)}{\partial f_s^l} > 0\right)$. *There are no fire sales in the high state of nature* $\left(f_s^h = f_p^h = 0\right)$. *In the low state, shareholders' fire sales are excessive both in absolute terms* $\left(f_p^l < f_s^l\right)$ *and per unit of debt* $\left(\frac{f_s^l}{d_s} > \frac{f_p^l}{d_p}\right)$. *As a consequence, asset prices are over-depressed relative to the optimum* $\left(q_p^l > q_s^l\right)$, *and shareholders overborrow* $(d_s > d_{p.})$.

Fire sales happen at a lower price than the asset's NPV. Thus, the bank only sells when short-term debt generates losses (i.e., in the low state of nature). It is useful to think of fire sales as the reaction to a negative liquidity shock. The optimal level of fire sales is determined by

$$1 + \lambda_j^l = \frac{R^{-1}b}{\left(\frac{\partial q(f_j^l) f_j^l}{\partial f_j^l}\right)}, \text{ for } j = p, s. \tag{10}$$

The right hand side is the ratio between the discounted long-term marginal revenue of the asset, $R^{-1}b$, and the marginal short-term revenue obtained from a fire sale, $\frac{\partial q(f^l) f^l}{\partial f^l}$. This ratio is the *loss ratio* from a fire sale. It is larger than one because, in the low state, when the liquidity shock triggers short-term losses, both shareholders and the planner need to sell assets at a loss to avoid negative profits. The variable $\lambda^l > 0$ is the Lagrange multiplier of equation (4) in the low state; and it captures the value of liquidity. The larger λ^l is, the larger the value the bank places on receiving one unit of revenue to keep its profits non-negative. Hence, as λ^l increases, the bank is willing to sell assets at a higher discount.

The value of liquidity in equilibrium is related to the optimal debt choice through the ratio between the expected gains from short-term debt in the high state and the expected losses in the low state:

$$1 + \lambda^l = \frac{p(a^h - R)}{(1-p)(R - a^l)} > 1. \tag{11}$$

This ratio is larger than one to compensate for the possibility of fire sales. At the optimal level of fire sales, the shareholders and the planner equate the ratio of losses from a fire sale to the ratio of gains from debt:

$$\frac{R^{-1}b}{\left(\frac{\partial q(f_j^l)f_j^l}{\partial f_j^l}\right)} = \frac{p(a^h - R)}{(1-p)(R - a^l)}. \tag{12}$$

The inefficiency comes from the left-hand side of equation (12). The shareholders and the planner differ in how they compute the revenue from fire sales. The planner takes into consideration the price effects of her fire sales:

$$\frac{\partial q(f_p^l)f_p^l}{\partial f_p^l} = q(f_p^l) + \frac{\partial q(f_p^l)}{\partial f_p^l} f_p^l.$$

while shareholders assume that they can sell assets at a given constant price:

$$\frac{\partial q(f_s^l)f_s^l}{\partial f_s^l} = q(f_s^l).$$

Since $\frac{\partial q(f_p^l)}{\partial f_p^l} < 0$ for all f^l, shareholders overestimate their revenues from fire sales. As a consequence, shareholders over-sell both in absolute terms, $f_p^l < f_s^l$, and per unit of debt, $\frac{f_s^l}{d_s} > \frac{f_p^l}{d_p}$. Consequently, shareholders' fire sale prices are lower than the planner's prices, $q_p^l > q_s^l$. Shareholders' behavior is suboptimal, as $\Pi_p - \Pi_s > 0$. In other words, shareholders sell too many long-term assets.

Using $\pi_1^l(d, f^l) = 0$, we can express the difference in levels of debt as:

$$d_s - d_p = \frac{\left(q_s^l f_s^l - q_p^l f_p^l\right)}{R - a^l}.$$

Shareholders overborrow if the revenue from fire sales is increasing in the amount of fire sales;

that is, when $\frac{\partial q(f^l)f^l}{\partial f^l} > 0$. This condition is equivalent to saying that the price elasticity of asset demand, $\xi(q) = -\frac{\partial f(q)}{\partial q}\frac{q}{f(q)}$, is larger than one. Since the planner understands equation (1) she is always in that elasticity range (otherwise she could decrease her fire sales and increase revenue). Parameter restriction (8) ensures that shareholders are also in that region of the demand elasticity. Relaxing this assumption does not alter the result that shareholders are inefficient; on the contrary, shareholders would be even more inefficient because revenue falls as sales increase when $\xi(q) < 1$.

4 Compensation Structures

We now assume the bank is managed by risk-neutral executives who do not internalize equation (1) and maximize the expected present value of their compensation:

$$\begin{aligned} &\underset{d_m, f_m^s}{max}\mathbb{E}\left(F_1 + S_1(\pi_1^s, \pi_2^s) + V_1(\pi_1^s) + R^{-1}\left(F_2 + V_2(\pi_2^s)\right)\right) \\ &s.t.\ (4) - (7) \end{aligned} \tag{13}$$

where F_1 and F_2 are fixed payments in each period; $S_1(\pi_1, \pi_2)$ is the equity (shares) compensation valued at period $t = 1$; and $V(\pi_1)$ and $V_2(\pi_2)$ are the variable bonuses payable, respectively, in period $t = 1$ and $t = 2$. Only the variable components (equity and bonus) are relevant for the executives' choices. We compare four compensation schemes: equity, deferred equity, long-term bonus and bail-in bonds.

4.1 Compensation Structure #1: Executives Paid with Equity, Deferred Equity and Long-term Bonuses

Bank executives are paid $\theta_1 \geq 0$ shares in period 1, and a percentage $\theta_2 \geq 0$ of period 2 profits (a long-term bonus). Additionally, they are awarded $\theta_1^d \geq 0$ of deferred shares. That is, executives have the right to θ_1^d of period 1 profits that can be cashed in period 2. We normalize, without loss of generality, the number of outstanding bank shares to 1. Hence, θ_1 and θ_1^d can be understood, respectively, as the number of shares and deferred shares granted to the bank's executives. Thus, the variable payoffs are:

$$S_1(\pi_1^s, \pi_2^s) = \theta_1(\pi_1^s + R^{-1}\pi_2^s) + \theta_1^d R^{-1}(\pi_1^s + \pi_2^s), \tag{14}$$

$$V_2(\pi_2^s) = \theta_2\pi_2^s, \tag{15}$$

for $s = \{h, l\}$, subject to the restriction that executives can never attain more than 100% of the company's profits each period:

$$0 \le \theta_1 + \theta_1^d \le 1, \qquad (16)$$
$$0 \le \theta_1 + \theta_1^d + \theta_2 \le 1.$$

There is no bonus in period 1 ($V_1(\pi_1^s) = 0$). The following proposition summarizes our results. We use subindex me to denote executives' decisions when offered equity payments.

Proposition 2 *Deferred shares and long-term bonuses reduce the fire sale inefficiency relative to the inefficiency which arises when the bank is managed directly by the shareholders. In particular, if* $\left[\theta_1^d (1 - R^{-1}) + \theta_2\right] > 0$ *then* $q_{me}^l > q_s^l$; $f_{me}^l < f_s^l$; $\frac{f_s^l}{d_s} > \frac{f_{me}^l}{d_{me}}$; *and* $(\Pi_{me} - \Pi_s) > 0$. *The ratio of debt is* $\frac{d_{me}}{d_s} = \frac{x\left(1 - \left(\frac{(1-p)(R-a^l)}{p(a^h-R)}\right)x\right)}{1 - \left(\frac{(1-p)(R-a^l)}{p(a^h-R)}\right)}$ *with* $x = \frac{\theta_1 + \theta_1^d + \theta_2}{\theta_1 + R^{-1}\theta_1^d}$. *The optimal level of fire sales* $\left(q_{me}^l = q_p^l;\ f_{me}^l = f_p^l\right)$ *can be achieved if* $\frac{\theta_1 + \theta_1^d + \theta_2}{\theta_1 + R^{-1}\theta_1^d} = \frac{1}{2}\left(\frac{p(a^h-R)}{(1-p)(R-a^l)} + 1\right) > 1.$

To see the intuition behind the results, we rewrite the first order conditions relative to fire sales and debt as:

$$\theta_1 + R^{-1}\theta_1^d + \lambda_{me}^l = \frac{(\theta_1 + \theta_1^d + \theta_2)R^{-1}b}{q_{me}^l}, \qquad (17)$$

$$\theta_1 + R^{-1}\theta_1^d + \lambda_{me}^l = \frac{(\theta_1 + R^{-1}\theta_1^d)p(a^h - R)}{(1-p)(R - a^l)}, \qquad (18)$$

where λ_{me}^l is the Lagrange multiplier of equation (4) in the low state of nature. It captures the value of liquidity. There are two key differences between this proposition and Proposition 1. First, the loss ratio from fire sales differs because the opportunity cost of a fire sale is the product of the long-term assets' NPV in period 1, $R^{-1}b$, and the share of long-term profits that accrue to the executives, $(\theta_1 + \theta_1^d + \theta_2)$. Second, the expected gains from short-term debt in the high state are weighted by the share of short-term profits belonging to the executives, $(\theta_1 + R^{-1}\theta_1^d)$. As in equation (12), at the optimal level of fire sales, the ratio of losses from a fire sale equates the ratio of gains from debt:

$$\frac{(\theta_1 + \theta_1^d + \theta_2)R^{-1}b}{q_{me}^l} = \frac{(\theta_1 + R^{-1}\theta_1^d)p(a^h - R)}{(1-p)(R - a^l)}. \qquad (19)$$

Comparing (12) and (19), we see that long-term bonus compensation, $\theta_2 > 0$, increases the executives' opportunity cost from a fire sale, which leads to a lower level of fire sales. Deferred compensation, $\theta_1^d > 0$, alters both the opportunity cost and the benefits from a fire sale. Given that executives face a positive discount factor, $R > 1$, the net effect of an increase in θ_1^d is to make fire sales more costly, ultimately reducing the executives' optimal level of fire sales. While the efficiency gain from long-term bonus compensation is independent of the discount rate, deferred compensation is less effective when the discount rate decreases (i.e., as R goes to 1). Thus, deferred compensation should never be placed in an account that pays an interest equal to or higher than the executives' discount rate. The socially efficient level of fire sales can be achieved when

$$\frac{\theta_1 + \theta_1^d + \theta_2}{\theta_1 + R^{-1}\theta_1^d} = \frac{1}{2}\left[1 + \frac{p\left(a^h - R\right)}{(1-p)\left(R - a^l\right)}\right] > 1. \tag{20}$$

That is, long-term compensation has to be big enough ($\theta_2 > 0$) or, if deferred compensation is used, the discount rate R has to be large enough. Notice that necessary conditions to restore efficiency are that either deferred shares are granted ($\theta_1^d > 0$), or a long-term bonus is used ($\theta_2 > 0$).

Equation (20) implies a multiplicity of efficient equilibria for different combinations of the compensation variables θ_1, θ_1^d, and θ_2. In Figure 1, we study numerically the implications of Proposition 2 using a comparative statics analysis. Table 1 contains the benchmark parameters. With these parameter values, the socially optimal ratio in the right hand-side of equation (20) equals 1.26. This is represented by the dotted line in the three graphs at the top row in Figure 1. Let us discuss the left column first. We set the compensation parameters $\theta_1^d = 0.05$ and $\theta_2 = 0.1$ and plot the ratio in the left hand-side of equation (20) as the number of (plain-vanilla) shares θ_1 changes. Solving for θ_1 in the efficiency condition (20), and keeping the other two variables constant, we see that this ratio crosses the socially efficient value at $\theta_1 = 0.347$. Notice that, if $\theta_1^d = \theta_2 = 0$, the equation has no solution. In other words, plain-vanilla shares cannot restore social efficiency *per se*. Share prices are calculated as the present value of payoffs from both period 1 and 2. The number of plain-vanilla shares granted does not affect the relative weight of short-term (period 1) versus long-term (period 2) compensation for the executives. Hence, shares cannot be used to overweight period 2 losses from fire sales relative to short-term (period 1) gains from leverage.

Table 1: Parameters

$v = 0.1$	$b = 1.05$	$R = 1.05$
$p = 0.82$	$k_0 = 11$	$a^h = 1.05$
$a^l = 0.9$	$\theta_1^d = 0.05$	$\theta_1 = 0.1$
$\theta_2 = 0.1$	$\gamma_2 = 0.1$	$\phi = 0.1$

The example shows that, if other time-contingent compensation structures are used in combination with shares, efficiency can be restored. In that case, the number of shares granted has welfare implications. At $\theta_1 = 0.347$, the executives' short-term leverage (second row graph) and the level of fire sales (third row graph) coincide with those of the socially efficient planner. As θ_1 increases above 0.347, the executives over-borrow and fire sales are excessive. Consequently, as plotted in the bottom graph, in the event of fire sales, the long-term asset price depreciates relative to the price which corresponds to the efficient level of fire sales. Notice that as θ_1 decreases, this price increases (see equation 1). For $\theta_1 < 0.347$, the asset could be sold in period 1 at a price higher than the planner's optimal price. This is, however, inefficient. In other words, minimizing the asset price discount in the event of fire sales may lead to a socially suboptimal outcome. Given the parameter values, the executives must be granted enough shares to induce an optimal level of leverage. In our example, any number of shares below 0.347 would render the executives over-cautious, relative to the socially optimal risk-taking level. The executives would not exploit all the return-enhancing possibilities available through short-term debt. This simple example illustrates the unintended negative consequences of regulating the maximum level of equity compensation.

In the middle column of Figure 1 we perform a similar exercise: keeping $\theta_1 = 0.2$ and $\theta_2 = 0.1$, we change the amount of deferred shares, θ_1^d. In this case, the socially efficient level of debt and fire sales is attained at $\theta_1^d = 0.241$. The comparative statics analysis is qualitatively analogous to those analyzed in the case of θ_1.

In the right column, we present a comparative statics analysis for the long-term bonus. Notice that the slopes are symmetric with respect to the previous two cases. That is, given $\theta_1 = 0.1$ and $\theta_1^d = 0.05$, the optimal compensation ratio is achieved for $\theta_2 = 0.061$. The higher this component is, relative to share compensation, the higher the opportunity cost of fire sales. Hence, increasing θ_2 decreases both leverage and fire sales, ultimately increasing the asset price in the event of fire sales. However, for values of θ_2 above the efficient level, the bank under-borrows and fire sales are below the socially efficient level. In that case, bank executives do not have sufficient incentives to efficiently exploit short-term leverage.

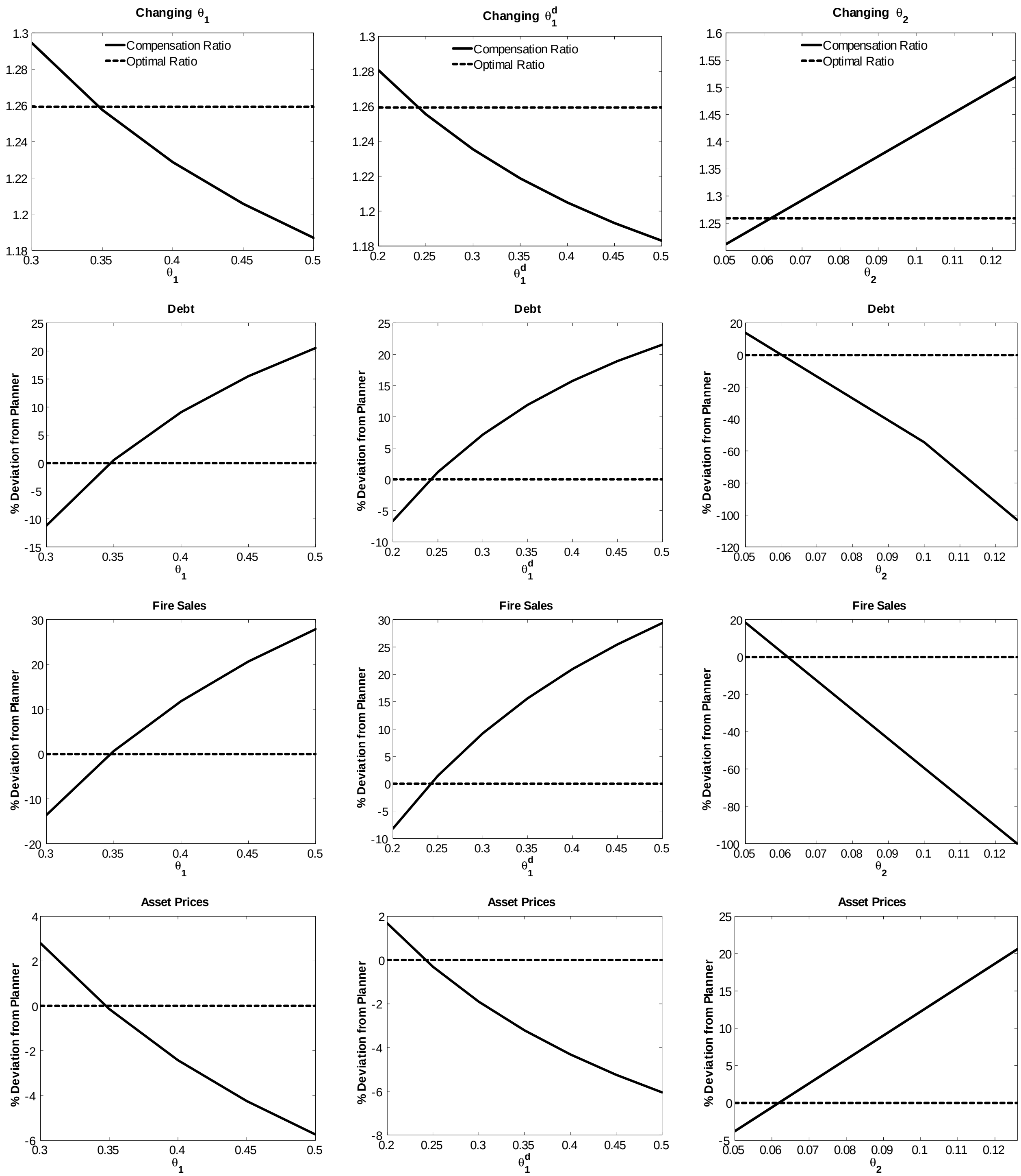

Figure 1: Comparing the Planner with Executives Paid with Equity. This figure

analyzes Proposition 2 numerically. The first row shows how the compensation ratio $\frac{\theta_1+\theta_1^d+\theta_2}{\theta_1+R^{-1}\theta_1^d}$ moves relative to the optimal ratio, $\frac{1}{2}\left[\frac{p(a^h-R)}{(1-p)(R-a^l)}+1\right]$. The second row plots the executives' optimal choice of debt as a percentage deviation from the planner's choice. The third row is the executives' choice of fire sales as a percentage deviation from the planner's choice. The last row represents the percentage deviation (with respect to the planner's outcome) of the asset's price in the event of fire sales as the executives' compensation changes. Parameter values are in Table 1. In the left column, only first period shares, θ_1, change. In the middle column, only deferred shares, θ_1^d, change. Finally, in the right column, only second period shares, θ_2, vary.

4.2 Compensation Structure #2: Bail-in bonds

Bail-in bonds allow us to implement state-contingent compensation. That is, the return for the executives of one unit of profits in period 1 can differ in the high and low states of nature. Under this compensation scheme, executives will receive a bonus related to the profits of period 1, $B_1(\pi_1)$, but in the low state of nature the executives' bonus is converted into equity:

$$V_1(\pi_1)=\begin{cases} B_1(\pi_1) & \text{if } s=h,\\ \gamma_1\pi_1 & \text{if } s=l.\end{cases}$$

Similarly, in the second period:

$$V_2(\pi_2)=\begin{cases} B_2(\pi_2) & \text{if } s=h,\\ \gamma_2\pi_2 & \text{if } s=l,\end{cases}$$

subject to the restrictions $\gamma_1+\gamma_2\leq 1$, $B_1(\pi_1)<\pi_1$ and $B_2(\pi_2)<\pi_2$. To focus on the effect of contingent bail-in bonds, we assume no stock is granted to the executives; that is, $S_1=0$. Our results are summarized in the next proposition. We use subindex mb to denote executives' decisions when paid with bail-in bonds. $B_1'(\pi_1)$ denotes the derivative.

Proposition 3 *The optimal level of fire sales* $\left(q_{mb}^l=q_p^l \text{ and } f_{mb}^l=f_p^l\right)$ *can be achieved if* $\frac{\gamma_2}{B_1'(\pi_1)}=\frac{1}{2}\left[1+\frac{p\left(a^h-R\right)}{(1-p)\left(R-a^l\right)}\right]$. *Moreover, if* $\frac{\gamma_2}{B_1'(\pi_1^h)}>1$, *then* $q_{mb}^l>q_s^l$; $f_{mb}^l<f_s^l$; $\frac{f_s^l}{d_s}>\frac{f_{mb}^l}{d_{mb}}$; *and* $(\Pi_{mb}-\Pi_s)>0$. *Also,* γ_1 *and* $B_2(\pi_2)$ *play no role in the executives' choices.*

The first order conditions relative to fire sales and debt in the low state are

$$\gamma_1 + \lambda_{mb}^l = \frac{\gamma_2 R^{-1} b}{q_{mb}^l}, \tag{21}$$

$$\gamma_1 + \lambda_{mb}^l = \frac{B_1'(\pi_1^h) p (a^h - R)}{(1-p)(R - a^l)}, \tag{22}$$

where λ_{mb}^l is the Lagrange multiplier of equation (4) in the low state of nature. Relative to Proposition 1, we observe two key differences: 1) An increase in γ_2 increases the opportunity cost of fire sales for executives, hence reducing their incentives toward them; 2) The derivative $B_1'(\pi_1^h)$ controls the marginal gain from debt in the high state for the executives. Reducing $B_1'(\pi_1^h)$ is similar to taxing the rewards from debt. This mechanism also reduces fire sales. To achieve optimality, we need the marginal change of the first period bonus not to be too large relative to the share of ownership in the second period:

$$\frac{\gamma_2}{B_1'(\pi_1)} = \frac{1}{2}\left[1 + \frac{p\left(a^h - R\right)}{(1-p)\left(R - a^l\right)}\right] > 1. \tag{23}$$

Figure 2 studies numerically the results from Proposition 3. For simplicity, we assume $B_1'(\pi_1^h)$ is constant and equal to ϕ. The values of the parameters are the same as in Table 1. The first column plots ϕ for a given γ_2, while the second column plots γ_2 given ϕ constant. The first row shows how the compensation ratio, $\frac{\gamma_2}{B_1'(\pi_1)}$, in the left hand-side of equation (23) moves relative to the optimal ratio, $\frac{1}{2}\left[1 + \frac{p}{(1-p)}\frac{\left(a^h - R\right)}{\left(R - a^l\right)}\right]$, in the right hand-side. When the ratios intersect (at value 1.26, given the assumed parameter values) both the executives and the planner achieve the same results and constrained efficiency is restored. In the left column, for $\gamma_2 = 0.1$, this is achieved at $\phi = 0.08$. As ϕ increases, the rewards from period 1 debt grow. Hence, debt and fire sales increase and, in the event of fire sales, the asset price decreases relative to the socially optimal price. If the rewards are too small (namely, for $\phi < 0.08$) the executives do not have enough short-term incentives for an efficient level of leverage in period 1. In that case, we observe both sub-optimal leverage and sub-optimal fire sales.

In the right column, we plot the comparative statics analysis for γ_2, keeping $\phi = 0.1$ constant. The socially efficient level of fire sales is restored at $\gamma_2 = 0.126$. When the long-term bonus is not large enough ($\gamma_2 < 0.126$), executives over-leverage, and fire sales are higher than the socially optimal level. The reverse happens when the executives' long-term incentives are above the planner's optimal choice. Excessive regulation on the minimum size of long-term bonuses may lead to under-leverage relative to the socially optimal level.

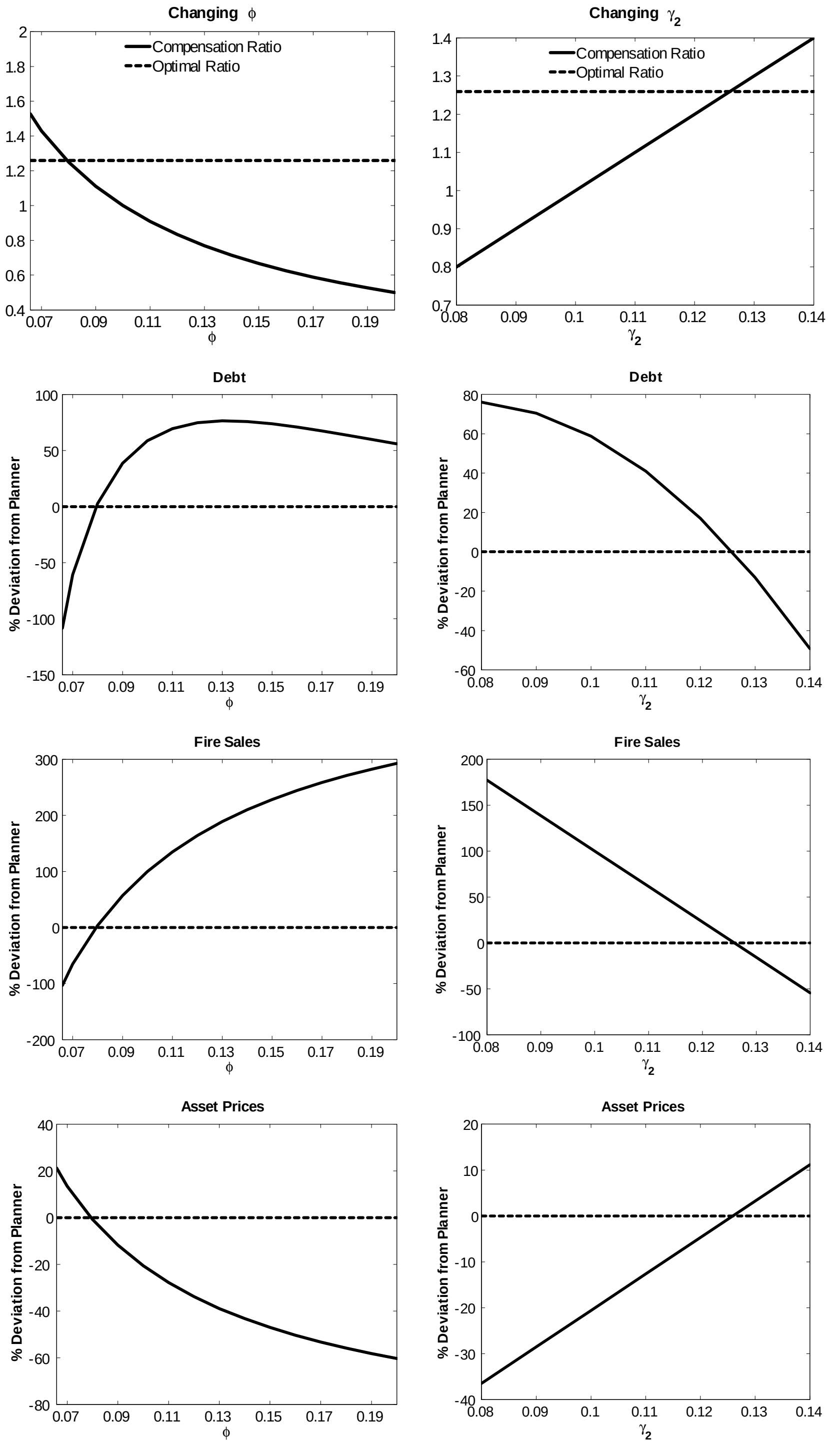

Figure 2: Comparing the Planner with Executives Paid with Bail-in bonds. This figure analyzes Proposition 3 numerically. The first row shows how the compensation ratio $\frac{\gamma_2}{B_1'(\pi_1)}$ moves relative to the optimal ratio $\frac{1}{2}\left[1+\frac{p(a^h-R)}{(1-p)(R-a^l)}\right]$. The second (third) row plots the executives' optimal choice of debt (fire sales) as a percentage deviation from the planner's choice. The last row

represents the percentage deviation (with respect to the planner's outcome) of the asset price in the event of fire sales as the executives' compensation changes. The derivative of the bonus with respect to profit in the first period $B_1'(\pi_1)$ is assumed to be constant and equal to ϕ. In the left column, ϕ varies while γ_2 is 0.1; in the right column, γ_2 changes while ϕ is 0.1.

4.3 Comparing the Compensation Structures

Deferred shares, long-term bonuses and bail-in bonds achieve efficiency if they satisfy (20) and (23). Now, we compare the compensation structures by evaluating which schedule is less expensive to implement. That is, which structure demands the lowest level of payments to the executives to achieve the planner's efficient allocation? We define the expected payments to the executives paid with (deferred) equity and long-term bonuses as follows:

$$W_{me} = p\left[\theta_1\pi_1^h + R^{-1}\left(\theta_1^d\pi_1^h + \left(\theta_1 + \theta_1^d + \theta_2\right)\pi_2^h\right)\right] + (1-p)\left[\theta_1\pi_1^l + R^{-1}\left(\theta_1^d\pi_1^l + \left(\theta_1 + \theta_1^d + \theta_2\right)\pi_2^l\right)\right].$$

Similarly, for the executives paid with bail-in bonds, the expected pay is written as follows:

$$W_{mb} = p\left[B_1(\pi_1^h) + R^{-1}B_2(\pi_2^h)\right] + (1-p)\left[\gamma_1\pi_1^l + R^{-1}\gamma_2\pi_2^l\right].$$

Proposition 4 summarizes our results.

Proposition 4 *It is always possible to find a scheme based only on bail-in bonds in which $B_2(\pi_2^h) = 0$. This scheme is cheaper to implement than a scheme involving long-term bonuses and/or deferred equity. That is, $W_{me} > W_{mb}$.*

Bail-in bonds compose a more efficient compensation structure because they are state dependent. That is, they provide incentives only when they are needed. This means that, for a socially optimal level of incentives, it is always possible to set the high state compensation equal to zero in period 2, $B_2(\pi_2^h) = 0$. In other words, with bail-in contingent bonds, shareholders can reduce the executives' compensation to the minimum in states without liquidity needs. In contrast, long-term bonuses, θ_2, are "blind" tools that pay in both states.

5 Explicit Modeling of the Inefficiency

In this section, we introduce a mechanism that makes fire sales inefficient. We follow the mechanism proposed by Krugman (1998), which allows for closed-form solutions. Fire sales are inefficient if the assets end up in the hands of buyers who mismanage them. For example, Krugman (1998) complained about Michael Jackson's purchase of a ski resort from a distressed South Korean bank.

We assume the economy has two sets of agents. On one side is the bank, who efficiently manages long term assets. On the other side is a representative unskilled investor who pays a quadratic cost $\frac{1}{2}vx^2$, with $v \geq 0$, to manage x units of long-term assets. For example, these costs can be inferior management or informational skills.[5] For simplicity, this investor has no initial endowment of long-term assets and cannot invest in short-term debt. We use ˜ to denote the variables of the unskilled investor. Her profits are

$$\tilde{\pi}_1^s(\tilde{f}^s) = -q\tilde{f}^s - \frac{1}{2}v\left(\tilde{f}^s\right)^2, \tag{24}$$

$$\tilde{\pi}_2^s(\tilde{f}^s) = \tilde{f}^s b. \tag{25}$$

That is, in period 1, the unskilled investor buys long-term assets from the bank and pays the costs of managing or understanding them. In the second period, she receives payments from the assets. The unskilled investor solves

$$\tilde{\Pi} = \max_{\tilde{f}^s} \mathbb{E}\left(\tilde{\pi}_1^s + R^{-1}\tilde{\pi}_2^s\right),$$

subject to $\tilde{f}^s \geq 0$. The FOC from this problem yields the price function (1).

In equilibrium, market clearing implies that the assets bought by the unskilled investor equal the assets sold by the bank; that is:

$$\tilde{f}^s = f^s, \forall s = l, h. \tag{26}$$

The set of feasible allocations is formally defined as follows:

Definition 1 *Let $F = \left\{d, f^h, f^l, \tilde{f}^h, \tilde{f}^l\right\}$ denote the set of feasible allocations such that restrictions (4)-(7), and the market clearing condition (26) hold.*

We define the utility of the bank net of transfers:

$$U_B\left(d, f^s, T^s\right) = E\left(\pi_1^s + R^{-1}\pi_2^s + T^s\right),$$

[5] As anecdotal motivation, the Financial Times (2012) reported that many funds interested in buying mortgages from Spanish banks incurred significant costs to understand and assess their values.

and the utility of the unskilled investor net of transfers:

$$U_U\left(\tilde{f}^s, \tilde{T}^s\right) = E\left(\tilde{\pi}_1 + R^{-1}\tilde{\pi}_2 + \tilde{T}^s\right).$$

Then, Pareto allocations are those levels of debt and fire sales that maximize total output. This total output can then be redistributed with a system of transfers:

Definition 2 *$P \subset F$ denotes the set of Pareto allocations. That is, for all allocations $x = \left\{d, f^h, f^l, \tilde{f}^h, \tilde{f}^l\right\} \in P$ there is no other allocation $x' \in F$ for which a system of transfers $\left\{T_t^s, \tilde{T}_t^s\right\}$ in zero net supply $\left(T_t^s + \tilde{T}_t^s = 0,\ \forall t,\ \forall s\right)$ exists such that $U_B(x') \geq U_B(x)$, $U_U(x') \geq U_U(x)$ with at least one the previous inequalities being strict inequality.*

The next Proposition shows that the planner's allocations are indeed the set of Pareto allocations.

Proposition 5 *An allocation $x = \left\{d, f^h, f^l, \tilde{f}^h, \tilde{f}^l\right\}$ is Pareto optimum if and only if whoever makes the bank's decisions internalizes the price function (1).*

The intuition for the previous result is that the inefficiency manifests itself in the resources the unskilled investor wastes when she acquires long-term assets $(\upsilon > 0)$. Pareto allocations minimize that waste of resources. However, "minimizing" the waste does not eliminate such waste entirely, since such a result is only possible when debt is zero. But zero debt is not optimal, since the expected return from debt is positive. The optimal result is for the bank's manager to select the right level of debt while taking into account social losses from fire sales in the bad state of nature. Optimal policy chooses the right level of debt by correctly internalizing the costs of the fire sales associated with debt.

6 Risk Aversion

In this section, we relax the risk-neutrality assumption to analyze how that would alter our results. First, we study a mean-variance maximization approach to focus on analytical results. Then, we present numerical results from an expected utility maximization approach. Both approaches deliver the same result: the higher the risk aversion of shareholders and executives, the lower their willingness to take on the risk associated with short-term debt. Lower leverage implies fewer fire-sales in the low state of nature. In other words, higher risk aversion reduces overborrowing and the need for regulating executive compensation.

First, we analyze a risk-averse bank shareholder with mean-variance utility. She solves:

$$\Pi_\alpha = \max_{d,f^s} \mathbb{E}\left(\pi_1 + R^{-1}\pi_2\right) - \frac{\alpha}{2} Var\left(\pi_1 + R^{-1}\pi_2\right), \tag{27}$$

subject to equations (4)-(7). The shareholder does not internalize the general equilibrium effects of her fire sales; that is, she ignores (1). The parameter $\alpha > 0$ denotes the shareholder's absolute risk aversion coefficient. We define

$$\Delta_s = \pi_1^h(d_s, f_s^h) + R^{-1}\pi_2^h(d_h, f_s^h) - \left(\pi_1^l(d_s, f_s^l) + R^{-1}\pi_2^l(d_l, f_s^l)\right), \tag{28}$$

which represents the difference between the profits in the high and low states of nature. The optimal level of fire sales in the low state is determined by

$$1 + \frac{\lambda_s^l}{1 + \alpha p^2 \Delta_s} = \frac{R^{-1}b}{q_s^l}. \tag{29}$$

This condition is equivalent to (10) for risk-neutral shareholders. We can interpret the equation in similar terms. The right-hand side is the *loss-ratio* from fire sales; that is, it represents the difference between what long-term assets pay in period 2 and what they would provide if sold in period 1. The left-hand side captures the willingness to sell assets at a loss. Similarly, we can derive

$$1 + \lambda_s^l = \frac{p(a^h - R)}{(1-p)(R - a^l)} - \alpha p \frac{a^h - a^l}{R - a^l}\Delta_s, \tag{30}$$

which is the risk-averse equivalent to (11). It shows the tradeoff between the gains of short-term debt and the expected losses in the low state. As the risk aversion coefficient, α, increases, the risk-averse shareholder values less the gains from short-term debt, which is a risky lottery (that is, there is variance in the payoffs) and is less willing to sacrifice long term assets at a high discount. Thus, risk aversion decreases leverage and fire sales in the bad state of nature. We solve for the level of fire sales for the risk-averse shareholder as follows:

$$q_s^l = R^{-1}b \frac{1 + \alpha p \Delta_s}{1 + \alpha p \Delta_s + \lambda_s^l}, \tag{31}$$

where λ_s^l is a function of q_s^l from condition (30) and $\Delta_s = (a^h - R)d_s + R^{-1}bf_s^l > 0$. If $\alpha = 0$, then (31) becomes the risk neutral close-form expression in (A17). At higher levels of risk aversion, $\alpha > 0$, fire sales prices are higher, which, by (1), means that both fire sales and leverage are lower.

Similarly, we can analyze a risk-averse executive with mean-variance utility compensated with equity, deferred equity and long-term bonuses as described in equations (14) and (15).

Given the compensation structure, the risk-averse executive solves the following problem:

$$\begin{aligned}
&\max_{d_m, f_m^s} \mathbb{E}\left(F_1 + S_1(\pi_1^s, \pi_2^s) + V_1(\pi_1^s) + R^{-1}\left(F_2 + V_2(\pi_2^s)\right)\right) \\
&\quad - \frac{\alpha}{2} Var\left(S_1(\pi_1^s, \pi_2^s) + V_1(\pi_1^s) + R^{-1} V_2(\pi_2^s)\right) \\
&\quad s.t.\ (4) - (7).
\end{aligned}$$

If we define

$$\Delta_{me} = (\theta_1 + \theta_1^d R^{-1})(a^h - R) d_{me} + (\theta_1 + \theta_1^d + \theta_2) R^{-1} b f_{me}^l,$$

then we can show that, in the high state, there are no fire sales ($f_{me}^h = 0$). The executive's leverage choice is given by $d_{me} = \frac{q_{me}^l f_{me}^l}{(R-a^l)}$ where the price of fire sales, q_{me}^l, satisfies the following condition:

$$\theta_1 + \theta_1^d R^{-1} + \frac{\lambda_{me}^l}{1 + \alpha p \Delta_{me}} = \frac{(\theta_1 + \theta_1^d + \theta_2) R^{-1} b}{q_{me}^l}. \tag{32}$$

The optimal debt choice implies the following condition:

$$\theta_1 + R^{-1}\theta_1^d + \lambda_{me}^l = \frac{(\theta_1 + R^{-1}\theta_1^d) p (a^h - R)}{(1-p)(R - a^l)} - (\theta_1 + R^{-1}\theta_1^d) \alpha p \frac{a^h - a^l}{R - a^l} \Delta_{me}. \tag{33}$$

Combining (32) and (33), we obtain the implicit condition that q_{me}^l must satisfy:

$$q_{me}^l = R^{-1} b \frac{(\theta_1 + \theta_1^d + \theta_2)(1 + \alpha p \Delta_{me})}{(\theta_1 + R^{-1}\theta_1^d)(1 + \alpha p \Delta_{me}) + \lambda_{me}^l}, \tag{34}$$

where λ_{me}^l comes from equation (33). Notice that when $\alpha = 0$, equation (34) coincides with the risk-neutral condition (19). Again, risk aversion, $\alpha > 0$, reduces the executive's incentives for leverage and fire sales.

We now analyze a bank executive with mean-variance utility who is compensated with bail-in bonds as described in Section 4.2. Let us define $\Delta_{mb} = B_1'(\pi_1)(a^h - R) d_{mb} + \gamma_2 R^{-1} b f_{mb}^l + (B_2'(\pi_2) - \gamma_2) R^{-1} b k_0$. From the FOCs, the price of fire sales must satisfy the following condition:

$$\gamma_1(1 + \alpha p \Delta_{mb}) + \lambda_{mb}^l = \frac{\gamma_2 (1 + \alpha p \Delta_{mb}) R^{-1} b}{q_{mb}^l}. \tag{35}$$

From the executive's debt FOC, it follows that:

$$\gamma_1 + \lambda_{mb}^l = \frac{B_1'(\pi_1) p (a^h - R)}{(1-p)(R - a^l)} - \alpha p \left(B_1'(\pi_1) \frac{a^h - R}{R - a^l} + \gamma_1 \right) \Delta_{mb}. \tag{36}$$

Combining (35) and (36), we obtain the implicit condition that q^l_{mb} must satisfy:

$$q^l_{mb} = R^{-1}b\frac{\gamma_2(1+\alpha p\Delta_{mb})}{\gamma_1(1+\alpha p\Delta_{mb})+\lambda^l_{mb}}, \tag{37}$$

where λ^l_{mb} is defined in equation (36). Notice that when $\alpha = 0$, equation (37) coincides with the risk-neutral condition (A41). Again, higher risk aversion leads to lower leverage.

In the expected utility framework, the shareholder maximizes

$$\begin{aligned} &\max_{d,f^s} \mathbb{E}\left[u\left(\pi^s_1\right) + R^{-1}u\left(\pi^s_2\right)\right] \\ &s.t.\ (4)-(7), \end{aligned} \tag{38}$$

where $u\left(.\right)$ is a concave function. Figure 3 plots the constant absolute risk aversion (CARA) case; that is:

$$u\left(\pi\right) = \frac{1-e^{-\alpha\pi}}{\alpha}, \tag{39}$$

where α is the coefficient of absolute risk aversion. Figure 3 confirms that higher risk aversion leads to lower leverage and fewer fire sales.[6]

[6] The quantitative and empirical literatures studying fire sales externalities suggest that, for the observed levels of risk aversion, fire sales externalities are sizeable and socially costly. See for example Bianchi (2011).

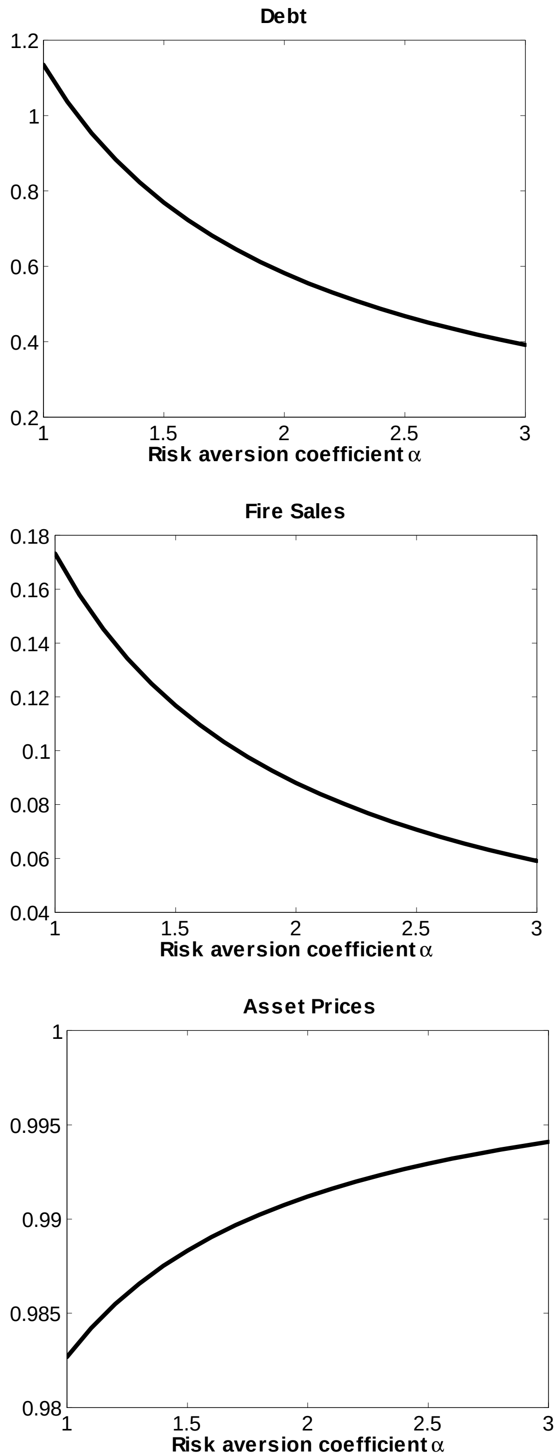

Figure 3: Risk Averse Shareholder. This figure reports the shareholder's choices when she is risk averse with CARA utility function, and maximizes expected utility. The plots are functions of the coefficient of risk aversion, α.

7 Government's Guarantees

The externalities analyzed above occur because banks do not internalize the general equilibrium effects of their actions when markets are incomplete. In this section, we consider a different rationale for regulation. We assume that banks enjoy an implicit, or explicit, government guarantee on their losses. That is, in the low state of nature (for instance, a financial crisis) the government steps-in to absorb some of the losses from short-term assets. We analyze how these guarantees affect our previous results.

We assume that, if bank's losses are $(R - a^l)d$, there is a government implicit guarantee such that $(1-\tau)\,\%$ of these losses are absorbed by the government, with $0 \leq \tau \leq 1$. That is, $\tau = 1$ is the case with no government guarantee, and $\tau = 0$ is the case in which the bank knows it would be completely bailed-out in the event of a crisis. With the government guarantee, the profit for banks in the first period of the low state becomes

$$\widehat{\pi}_1^l = \tau(a^l - R)\widehat{d} + \widehat{q}^l \widehat{f}^l. \tag{40}$$

We use ^to denote the variables when there are government guarantees. The next Proposition summarizes the results:

Proposition 6 *Government guarantees generate higher borrowings and fire sales. The levels of debt and fire sales selected by shareholders are increasing in the guarantees. That is,* $\frac{\partial \widehat{f}_s^l}{\partial \tau} < 0$ *and* $\frac{\partial \widehat{d}_s}{\partial \tau} < 0$*. The same is true for executives. Optimal regulation of executives paid with equity requires we set*

$$x(\tau) = \frac{\theta_1 + \theta_1^d + \theta_2}{\theta_1 + R^{-1}\theta_1^d} = \frac{1 + \sqrt{1 - \tau\left(1 - \left(\frac{(1-p)(R-a^l)}{p(a^h-R)}\right)^2\right)}}{2\tau \frac{(1-p)(R-a^l)}{p(a^h-R)}}, \tag{41}$$

with $\frac{\partial x(\tau)}{\partial \tau} < 0$.

Optimal regulation of the executives paid with bail-in bonds requires:

$$\frac{\gamma_2}{B_1'(\pi_1^h)} = \frac{1 + \sqrt{1 - \tau\left(1 - \left(\frac{(1-p)(R-a^l)}{p(a^h-R)}\right)^2\right)}}{\frac{2\tau(1-p)(R-a^l)}{p(a^h-R)}}, \tag{42}$$

with $\frac{\partial y(\tau)}{\partial \tau} < 0$*. That is, the optimal compensation ratio decreases with* τ*.*

The government guarantees generate moral hazard. If short-term debt generates profits, then the bank gets to keep them. However, losses are socialized in an amount determined by the τ parameter. The moral hazard reinforces overborrowing and fire sales externalities. Optimal regulation works exactly as discussed in Section 4. The regulator regulates executive compensation to reduce rewards from short-term debt and to make agents internalize the costs of fire sales. The differences are related to the τ parameter. The larger the guarantees, then larger is the moral hazard and the greater optimal compensation needs to be to combat the rewards from debt.

8 Conclusions

We have studied a model in which bank executives borrow and underestimate the price effect of their fire sales. This failure encourages executives to over-borrow in the short run (relative to the socially optimal level) and to participate in excessive fire sales in the event of a bad shock. The suboptimal level of fire sales depresses banks' asset prices below the price that would result if executives had correctly accounted for the price impact of fire sales. We also analyzed moral hazard from government guarantees, which also leads to overborrowing.

We have discussed the ability of four different compensation schemes to deliver socially optimal levels of debt and fire sales. Our results show that some level of contingency in time and/or states of nature must be introduced in the compensation in order to induce the socially optimal levels of debt and fire sales. The basic idea is to make excessive fires sales burdensome enough for executives by decreasing the present value of their leverage gains, as deferred equity does, or by increasing the opportunity cost of fire sales, as long-term bonuses do. State-contingent compensation, like bail-in bonds, has an additional advantage: this scheme allows for incentives to apply only when they are necessary, for example, in the event of a liquidity crush that may trigger inefficient fire sales. The state-contingent nature of bail-in bonds makes them a less onerous way to provide incentives relative to deferred shares or long-term bonuses.

Another implication of our findings is that imposing boundaries on executive compensation may be suboptimal. Imposing a regulation on the absolute level of compensation may render executives either overcautious, or too risk-hungry, which is inefficient in either case. The composition of variable compensation is more important than the level of compensation itself.

An area for future research would be to integrate our model of atomistic banks (banks that are individually so small that a change in action by a single bank does not alter the equilibrium), with the fact that there exist some banks that are "too big to fail". One way to do so would be to add collateral constraints. Even if banks are big, with incomplete markets, heterogeneous banks may rationally decide to ignore the effect of their actions on the constraints of other banks. This results in a pecuniary externality that may justify public intervention.

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Appendix

Proof of Proposition 1:

We introduce the notation for the Lagrange multipliers. The subscript p denotes the social planner although we omit it when there is no risk of confusion. Let $p\lambda_p^h \geq 0$ denote the probability weighted Lagrange multiplier for the non-negativity restriction on period 1 profits in the high state of nature (we use the multiplier $(1-p)\lambda_p^l \geq 0$ for the low state). Likewise, we denote $p\mu_p^h \geq 0$ the multiplier for the non-negativity restriction on period 2 profits when the state is high (we use the multiplier $(1-p)\mu_p^l \geq 0$ for the low state). Finally, we call $\rho_p \geq 0$ and $p\psi_p^h \geq 0$ the multipliers on the non-negativity restrictions on debt and state h fire sales (we use the multiplier $(1-p)\psi_p^l \geq 0$ for the low state).

Using the previous definitions, the planner's problem in Section 3 leads to the following Lagrangian function:

$$\begin{aligned}
L(d_p, f_p^s) = {} & p\left(\pi_1^h(d_p, f_p^h) + R^{-1}\pi_2^h(d_p, f_p^h)\right) + (1-p)\left(\pi_1^l(d_p, f_p^l) + R^{-1}\pi_2^l(d_p, f_p^l)\right) + \\
& + p\lambda_p^h\pi_1^h(d_p, f_p^h) + (1-p)\lambda_p^l\pi_1^l(d_p, f_p^l) + p\mu_p^h\pi_2^h(d_p, f_p^h) + \\
& + (1-p)\mu_p^l\pi_2^l(d_p, f_p^l) + p\psi_p^h f_p^h + (1-p)\psi^l f_p^l + \rho_p d.
\end{aligned}$$

The planner chooses $d_p \geq 0$ and $f_p^s \geq 0$, $s = h, l$, such that $\frac{\partial L}{\partial f_p^h} = \frac{\partial L}{\partial f_p^l} = \frac{\partial L}{\partial d_p} = 0$. The first order conditions are:

$$(1+\lambda_p^h)\left(q'(f_p^h)f_p^h + q(f_p^h)\right) + \psi_p^h = (R^{-1} + \mu_p^h)b, \tag{A1}$$

$$(1+\lambda_p^l)\left(q'(f_p^l)f_p^l + q(f_p^l)\right) + \psi_p^l = (R^{-1} + \mu_p^l)b, \tag{A2}$$

$$\rho_p + p(1+\lambda_p^h)(a^h - R) = (1-p)(1+\lambda_p^l)(R - a^l). \tag{A3}$$

Equations (A1) and (A2) state that, at the optimum, the marginal utility of fire sales in each state (the left hand-side) must be equal to the marginal cost of fire sales (the right hand-side). The utility of fire sales is the marginal revenue from an extra fire sale multiplied by one plus the Lagrange multiplier which captures the benefit of one extra unit of profits in period 1. The cost of the fire sale is the foregone revenue, $R^{-1}b$, that the asset would have paid in period 2 plus the Lagrange multiplier which captures the benefit of one extra unit of profits in period 2.

Equation (A3) states that, at the optimum, the marginal expected utility of debt in the high state h must be equal to the marginal expected disutility of debt in the low state l.

The shareholders' problem (we use the subscript s to identify it) coincides with the problem of the planner, except that shareholders do not internalize equation (1). Hence, for shareholders

$$q'(f^s) = 0 \text{ for } s = l, h.$$

In other words, shareholders take fire sale prices as given and independent of the volume of their fire sales. Thus shareholders' first order conditions are:

$$(1 + \lambda_s^h)q_s^h + \psi_s^h = (R^{-1} + \mu_s^h)b, \tag{A4}$$

$$(1 + \lambda_s^l)q_s^l + \psi_s^l = (R^{-1} + \mu_s^l)b, \tag{A5}$$

$$\rho_s + p(1 + \lambda_s^h)(a^h - R) = (1 - p)(1 + \lambda_s^l)(R - a^l). \tag{A6}$$

We can see that only equations (A4) and (A5) differ from the planner's first order condition. Given that $q'(f_p^s) < 0$ for $s = l, h$, those equations show that shareholders over-estimate the marginal revenue from fire sales.

The slackness conditions of the shareholders and planner's problem are:

$$\lambda^h(q(f^h)f^h + (a^h - R)d) = 0, \tag{A7}$$

$$\lambda^l(q(f^l)f^l + (a^l - R)d) = 0, \tag{A8}$$

$$\mu^h b(k_0 - f^h) = 0, \tag{A9}$$

$$\mu^l b(k_0 - f^l) = 0, \tag{A10}$$

$$\psi^h f^h = 0, \tag{A11}$$

$$\psi^l f^l = 0, \tag{A12}$$

$$\rho d = 0. \tag{A13}$$

Our first result is that, given $q(f) < R^{-1}b$ if $f > 0$, for both the shareholders and the planner, it is not optimal to sell the long-term asset in the high state:

$$f_p^h = f_s^h = f^h = 0.$$

The intuition for this result is that, in good times, there is no need to sell at a loss. This result can be proven by contradiction. If $f^h > 0$, given $R < a^h$, (A7) implies $\lambda^h = 0$ and (A11) implies $\psi^h = 0$. These results, together with (1) ensure equations (A1) and (A4) hold only if

$\mu^h < 0$, a contradiction. Therefore $f^h > 0$ cannot be a solution. In fact, if shareholders and the planner could short sell the long-term asset in the high state ($f^h < 0$), then they would reap a risk-less profit $q(f^h) - R^{-1}b = vf^h > 0$, unbounded in f^h. Thus, $f_p^h = 0$ is the only solution in state h. Given (A7) and (A9), $f_p^h = 0$ implies

$$\lambda_s^h = \lambda_p^h = \lambda^h = 0,$$
$$\mu_s^h = \mu_p^h = \mu^h = 0.$$

Plugging these values in (A1) and (A4), it follows that $\psi_p^h = \psi_s^h = 0$. From condition (A6) in the shareholders' problem and condition (A3) in the planner's problem, we can write the Lagrange multiplier of debt at the optimum as a function of λ^l as follows:

$$\rho_s = \rho_p = \rho = (1-p)(R-a^l) - p(a^h - R) + \lambda^l(1-p)(R-a^l) \geq 0.$$

The relation $\lambda^l = \lambda_s^l = \lambda_p^l$ will be proven later.

From the latter condition, given the parameter restriction (8), if $\lambda^l = 0$, then $\rho < 0$, which is a contradiction. Hence, $\lambda^l > 0$. If $\rho > 0$, by (A13), $d = 0$, and this leads to another contradiction given (8). To see this, consider that if $d = 0$ then $f^l = 0$ by (A8), and this implies $\mu^l = 0$ by (A10). Then, given (1), $f^l = \mu^l = 0$ implies that $\psi^l = -\lambda^l R^{-1} b$ in condition (A2), which contradicts $\lambda^l > 0$. It follows that $\rho = 0$ and $f^l > 0$, which implies $\psi^l = 0$ according to (A12).

Using the previous results together with the first order conditions for debt in equation (A3) for the planner, and equation (A6) for the shareholders, we can write the Lagrange multiplier of fire sales in the low state for both agents as follows:

$$\lambda^l = \lambda_s^l = \lambda_p^l = \frac{p(a^h - R)}{(1-p)(R-a^l)} - 1 > 0, \tag{A14}$$

From (A5) we can also obtain the Lagrange multiplier of fire sales in the low state for the shareholders

$$\lambda_s^l = \frac{R^{-1}b}{q_s^l} - 1. \tag{A15}$$

where we apply the result $\mu_s^l = 0$ which will be proven below using parameter restriction (9).

Similarly, for the planner, we obtain the following from (A2)

$$\lambda_p^l = \frac{R^{-1}b}{q'(f_p^l)f_p^l + q(f_p^l)} - 1, \tag{A16}$$

where we are using the result $\mu_p^l = 0$ to be proven below using parameter restriction (9). Condition (A16) differs from shareholders' equation (A15) because the planner is taking into account the effects of her fire sales on the revenue from a fire sale.

Combining (A14) with (A15), we obtain the unique price at which shareholders are willing to engage in fire sale in the low state:

$$\frac{R^{-1}b}{2} < q_s^l = R^{-1}b\frac{(1-p)(R-a^l)}{p(a^h-R)} < R^{-1}b, \tag{A17}$$

where the inequalities follow from condition (8). Similarly, combining (A15) with (A16) we obtain

$$R^{-1}b\frac{(1-p)(R-a^l)}{p(a^h-R)} = q'(f_p^l)f_p^l + q(f_p^l),$$

which, using shareholders price (A17), can be rewritten as

$$q_s^l = q'(f_p^l)f_p^l + q_p^l, \tag{A18}$$

where we used q_p^l as the notation for $q(f_p^l)$.

Equation (A18) proves our second result. Given (1), fire sales prices decrease when fire sales increase, $q'(f_p^l) < 0$. Thus, if $f_p^l > 0$ shareholders over-depress asset prices:

$$q_s^l < q_p^l.$$

This result, together with equation (1), implies our third result: shareholders over-sell in absolute terms:

$$f_p^l < f_s^l.$$

Plugging the shareholders' fire sale price (A17) into equation (1) , we obtain the amount of shareholders' fire sales:

$$\begin{aligned} f_s^l &= \frac{R^{-1}b - q_s^l}{v} \\ &= \frac{R^{-1}b}{v}\left(1 - \frac{(1-p)(R-a^l)}{p(a^h-R)}\right) < \frac{R^{-1}b}{2v}, \end{aligned} \tag{A19}$$

where the inequality follows from the parameter restriction (8). By a similar logic, plugging (1) and (A17) in the equation for the planner's fire sale price (A18) gives the amount of the planner's fire sales:

$$
\begin{aligned}
f_p^l &= \frac{R^{-1}b - q_s^l}{2v} && \text{(A20)}\\
&= \frac{R^{-1}b}{2v}\left(1 - \frac{(1-p)(R-a^l)}{p(a^h - R)}\right) < \frac{R^{-1}b}{2v}, && \text{(A21)}
\end{aligned}
$$

where the inequality follows from the parameter restriction (8). The planner's fire sales are exactly half the shareholders' fire sales, $\frac{f_s^l}{f_p^l} = 2$.

From (A19) and the parameter restrictions (8) and (9) we see that $f_s^l < k_0$, which, together with the slack condition (A10), imply $\mu_s^l = 0$. Since $f_s^l < k_0$ and $f_p^l < f_s^l$ then $f_s^l < k_0$, which implies $\mu_p^l = 0$.

From (1) and (A20) we obtain another way to rewrite the relationships between planner and shareholders' prices (A18),

$$
\begin{aligned}
q_p^l &= R^{-1}b - v f_p^l \\
&= \frac{R^{-1}b}{2} + \frac{q_s^l}{2}. && \text{(A22)}
\end{aligned}
$$

From (A22) and the definition of q_s^l in (A17), it follows that

$$\frac{R^{-1}b}{2} < q_p^l < R^{-1}b. \tag{A23}$$

The price elasticity of the demand function is $\xi(q) = -\frac{\partial f(q)}{\partial q}\frac{q}{f(q)} = \frac{q}{R^{-1}b - q}$. For any $q > \frac{R^{-1}b}{2}$, the supply curve is elastic, that is, $\xi(q) > 1$. In the region where the supply curve is inelastic, $\xi(q) < 1$, the revenue from fires sales decreases as fire sales increase. Given (A17) and (A23), the parameter restriction (8) guarantees that the prices at which shareholders and the planner sell are in the elastic region of the demand function.

Given $\lambda^l > 0$, and equation (A8), the fire sales price determines the ratio of fire sales per unit of debt

$$
\begin{aligned}
\frac{f_s^l}{d_s} &= \frac{(R - a^l)}{q_s^l}, && \text{(A24)}\\
\frac{f_p^l}{d_p} &= \frac{(R - a^l)}{q_p^l}. && \text{(A25)}
\end{aligned}
$$

From $q_s^l < q_p^l$ and equations (A24) and (A25) we obtain our fourth result: shareholders also over-sell in relative terms. More specifically, they oversell in terms of fire sales per unit of debt borrowed:

$$\frac{f_s^l}{d_s} > \frac{f_p^l}{d_p}.$$

Showing that shareholders are inefficient is equivalent to showing that the planner achieves larger profits, $\left(\Pi_p\left(f_p^h, f_p^l, d_p\right) - \Pi_s\left(f_s^h, f_s^l, d_s\right)\right) > 0$. From profit functions defined in (2) and (3), we can write the planner's profit as follows:

$$\Pi_p\left(f_p^h, f_p^l, d_p\right) = d_p\left(p(a^h - R) - (1-p)(R - a^l)\frac{R^{-1}b}{q_p^l}\right) + R^{-1}bk_0. \tag{A26}$$

Analogously, we can write the shareholders' profit as follows:

$$\Pi_s\left(f_s^h, f_s^l, d_s\right) = d_s\left(p(a^h - R) - (1-p)(R - a^l)\frac{R^{-1}b}{q_s^l}\right) + R^{-1}bk_0.$$

Replacing q_s^l by (A17) in the previous equation, we obtain that $\Pi_s\left(f_s^h, f_s^l, d_s\right) = R^{-1}bk_0$. Hence, the profit gap between the planner and the shareholders can be written as follows:

$$\Pi_p\left(f_p^h, f_p^l, d_p\right) - \Pi_s\left(f_s^h, f_s^l, d_s\right) = d_p\left(p(a^h - R) - (1-p)(R - a^l)\frac{R^{-1}b}{q_p^l}\right) > 0.$$

The gap is positive because the function in parentheses is monotonously increasing in q^l and $q_p^l > q_s^l$. With some algebra and the previous results, the previous equation can be written as:

$$\Pi_p\left(f_p^h, f_p^l, d_p\right) - \Pi_s\left(f_s^h, f_s^l, d_s\right) = \left\{\begin{array}{c} \frac{d_s}{2}\left(1 + \frac{1}{2}vf_s^l\right)\left(p(a^h - R) + (1-p)(a^l - R)\right) + \\ +(1-p)\frac{1}{2}f_s^l\left(\frac{3}{2}R^{-1}b - q_s^l\right) \end{array}\right\}. \tag{A27}$$

Given the parameter restrictions, this profit gap is increasing in f_s^l. That is, the larger the shareholders' fire sales, the larger the inefficiency.

Now, we can solve for d_s and d_p. From (A17), (A19) and shareholders' ratio (A24) we obtain shareholders' debt:

$$\begin{aligned} d_s &= \frac{q_s^l f_s^l}{(R-a^l)} \\ &= \frac{(R^{-1}b)^2}{v(R-a^l)} \frac{(1-p)(R-a^l)}{p(a^h-R)} \left(1 - \frac{(1-p)(R-a^l)}{p(a^h-R)}\right). \end{aligned} \tag{A28}$$

Similarly, using (1) and the planner's ratio (A25), we can solve for the planner's debt as a function of the revenue from fire sales:

$$d_p(f) = \frac{q(f_p^l) f_p^l}{(R-a^l)} = \frac{R^{-1} b f_p^l - v\left(f_p^l\right)^2}{R-a^l}.$$

By replacing (A21) in the latter equation, we obtain the optimal debt choice of the planner as a function of the primitive parameters:

$$d_p = \frac{(R^{-1}b)^2}{4v(R-a^l)} \left(1 - \left(\frac{(1-p)(R-a^l)}{p(a^h-R)}\right)^2\right).$$

In order to compare the debt choices of the planner and the shareholders, we can rewrite the difference between both debt levels as follows:

$$\begin{aligned} d_s - d_p &= \frac{1}{R-a^l}\left(q_s^l f_s^l - q_p^l f_p^l\right) \\ &= \frac{1}{R-a^l}\left(q_s^l (f_s^l - f_p^l) - f_p^l (q_p^l - q_s^l)\right) \\ &= \frac{f_p^l}{R-a^l}\left(2q_s^l - q_p^l\right). \end{aligned}$$

Given (A17) and (A22) we obtain

$$d_s > d_p \iff q_s^l > \frac{R^{-1}b}{3},$$

which holds since $q_s^l > \frac{R^{-1}b}{2}$. In other words, parameter restriction (8) ensures that shareholders and the planner are only selling at fire sale prices in the price elastic region of the demand curve, and it is enough to guarantee $d_s > d_p$. *QED.*

Proof of Proposition 2:

We use the same notation for the Lagrange multipliers as in Proposition 1. The Lagrangian function in this case changes relative to Proposition 1 because the executives' objective function is the discounted value of the executives' compensation. We assume that executives, like shareholders, do not internalize that the price of fire sales depends on the amount of fire sales. That is, the executives ignore equation (1).

Using the previous definitions, the executives' problem in (13) leads to the following first order conditions (the subscript me denotes the executives are paid with equity):

$$(\theta_1 + R^{-1}\theta_1^d + \lambda_{me}^h)q_{me}^h + \psi_{me}^h = \left((\theta_1 + \theta_1^d + \theta_2)R^{-1} + \mu_{me}^h\right) b, \tag{A29}$$

$$(\theta_1 + R^{-1}\theta_1^d + \lambda_{me}^l)q_{me}^l + \psi_{me}^l = \left((\theta_1 + \theta_1^d + \theta_2)R^{-1} + \mu_{me}^l\right) b, \tag{A30}$$

$$\rho_{me} + p(\theta_1 + R^{-1}\theta_1^d + \lambda_{me}^h)(a^h - R) = (1-p)(\theta_1 + R^{-1}\theta_1^d + \lambda_{me}^l)(R - a^l). \tag{A31}$$

and to the slackness conditions (A7)-(A13).

Following the same arguments as in Proposition 1, we can prove that there are no fire sales in the good state (i.e. $f_{me}^h = 0$). This implies $\lambda_{me}^h = \mu_{me}^h = 0$ and $\psi_{me}^h = (\theta_1^d(1-R^{-1})+\theta_2)R^{-1}b$.

Also, following the logic from Proposition 1, given parameter restrictions (8) and (9) we can show that $d_{me} > 0$ and $\rho_{me} = \psi_{me}^l = \mu_{me}^l = 0$. Thus, we use equation (A31) to derive

$$0 < \lambda_{me}^l = (\theta_1 + R^{-1}\theta_1^d)\left(\frac{p(a^h - R)}{(1-p)(R-a^l)} - 1\right). \tag{A32}$$

This is the equivalent of equation (A14) in shareholders and planner problem. Notice that, given the parameter restriction in (16), the shadow price of debt decreases relative to the shareholders case.

From (A30) we obtain another expression for the Lagrange multiplier of fire sales in the low state:

$$\lambda_{me}^l = \frac{\left(\theta_1 + \theta_1^d + \theta_2\right) R^{-1} b}{q_{me}^l} - \left(\theta_1 + R^{-1}\theta_1^d\right). \tag{A33}$$

Combining (A32) and (A33), we obtain the unique price at which the executives paid with equity are willing to participate in fire sales in the low state:

$$q_{me}^l = R^{-1} b \frac{\left(\theta_1 + \theta_1^d + \theta_2\right)(1-p)(R-a^l)}{\left(\theta_1 + R^{-1}\theta_1^d\right) p(a^h - R)}. \tag{A34}$$

By comparing (A34) with (A17), we observe that

$$q_{me}^{l} = \frac{q_s^l\left(\theta_1 + \theta_1^d + \theta_2\right)}{\theta_1 + R^{-1}\theta_1^d},$$

and $q_{me}^l = q_s^l$ only if $\frac{\left(\theta_1+\theta_1^d+\theta_2\right)}{\left(\theta_1+R^{-1}\theta_1^d\right)} = 1$. Thus we obtain our result:

$$\frac{\theta_1 + \theta_1^d + \theta_2}{\theta_1 + R^{-1}\theta_1^d} > 1 \Longrightarrow \left(1 - R^{-1}\right)\theta_1^d + \theta_2 > 0 \Longrightarrow q_{me}^l > q_s^l,$$

or

$$\text{if } \left(1 - R^{-1}\right)\theta_1^d > 0 \text{ or } \theta_2 > 0 \text{ then } q_{me}^l > q_s^l. \tag{A35}$$

Using (A35) and equation (1) we derive that deferred compensation or long-term compensation allows for a reduction in shareholders' inefficient fire sales:

$$\text{if } \left(1 - R^{-1}\right)\theta_1^d > 0 \text{ or } \theta_2 > 0 \text{ then } f_{me}^l < f_s^l$$

In fact, by incorporating the executives' fire sale price (A34) into equation (1), we can solve for the executives' optimal fire sales in the bad state:

$$0 < f_{me}^l = \frac{R^{-1}b}{v}\left(1 - \frac{(1-p)(R-a^l)}{p(a^h-R)}\frac{\left(\theta_1 + \theta_1^d + \theta_2\right)}{\left(\theta_1 + R^{-1}\theta_1^d\right)}\right).$$

Given $\lambda_{me}^l > 0$, and equation (A8), the fire sale price determines the ratio of fire sales per unit of debt:

$$\frac{f_{me}^l}{d_{me}} = \frac{(R-a^l)}{q_{me}^l}. \tag{A36}$$

Thus, given (A35), it follows that $\frac{f_s^l}{d_s} > \frac{f_{me}^l}{d_{me}}$.

The ratio (A36) and q_{me}^l from (A34) give the absolute level of debt:

$$d_{me} = \frac{\left(\theta_1 + \theta_1^d + \theta_2\right)}{\left(\theta_1 + R^{-1}\theta_1^d\right)}\frac{(R^{-1}b)^2}{v(R-a^l)}\frac{(1-p)(R-a^l)}{p(a^h-R)}\left(1 - \frac{(1-p)(R-a^l)}{p(a^h-R)}\frac{\theta_1 + \theta_1^d + \theta_2}{\theta_1 + R^{-1}\theta_1^d}\right).$$

To show that $(1 - R^{-1})\theta_1^d + \theta_2 > 0$ is a sufficient condition for $d_{me} < d_s$, we proceed as follows. Let us define $x = \frac{\theta_1+\theta_1^d+\theta_2}{\theta_1+R^{-1}\theta_1^d}$ and $\psi = \frac{(1-p)(R-a^l)}{p(a^h-R)}$, where $1/2 < \psi < 1$ follows from parameter restriction (8). Then, we can write the ratio between the executives and shareholders' debt as a function $h(x)$:

$$\frac{d_{me}}{d_s} = h(x) = \frac{x(1-\alpha x)}{1-\psi}. \tag{A37}$$

At $x^* = \frac{1}{2\alpha} < 1$, the function $h(x)$ reaches a maximum $h(x^*) = \frac{1}{4\alpha(1-\psi)} > 1$. For all $x > x^*$, the function is monotonically decreasing $\left(\text{i.e. } \frac{\partial h(x)}{\partial x} = \frac{1-2\alpha x}{1-\psi} < 0\right)$. Notice that $h(1) = 1$. Therefore, for any $x = \frac{\theta_1+\theta_1^d+\theta_2}{\theta_1+R^{-1}\theta_1^d} > 1$, $h(x) = \frac{d_{me}}{d_s} < 1$.

To show that $\theta_1^d > 0$ or $\theta_2 > 0$ reduces the inefficiency, notice that the bank's total profit can be written as follows:

$$\Pi_{me}\left(f_{me}^h, f_{me}^l, d_{me}\right) = d_{me}\left(p(a^h-R) + (1-p)(a^l-R)\frac{R^{-1}b}{q_{me}^l}\right) + R^{-1}bk_0.$$

Additionally, the profit gap relative to the case where the bank is run by the shareholders is:

$$\Pi_{me}\left(f_{me}^h, f_{me}^l, d_{me}\right) - \Pi_s\left(f_s^h, f_s^l, d_s\right) = d_{me}\left(p(a^h-R) + (1-p)(a^l-R)\frac{R^{-1}b}{q_{me}^l}\right).$$

The gap will be positive as long as $q_{me}^l > q_s^l$ or, according to our previous result, $\theta_1^d\left(1-R^{-1}\right)+$ $\theta_2 > 0$. Comparing q_{me}^l from (8) and q_p^l from (A22), we obtain the compensation scheme that allows to achieve the socially optimal level of fire sales:

$$q_{me}^l = q_p^l \text{ iff } \frac{\theta_1+\theta_1^d+\theta_2}{\theta_1+R^{-1}\theta_1^d} = \frac{1}{2}\left(\frac{p(a^h-R)}{(1-p)(R-a^l)}+1\right).$$

There are multiple combinations of θ_1^d and θ_2 that satisfy this restriction. The intuition is to have θ_2 large enough relative to θ_1, or $\theta_1^d > 0$ when executives face a large discount factor. *QED.*

Proof of Proposition 3:

We use the same notation for the Lagrange multipliers as in Proposition 1. Now, the Lagrangian function changes relative to Proposition 1 because the executives' objective function is the discounted value of equations $V_1(\pi_1^s)$ and $V_2(\pi_2^s)$. Again, we assume that the executives, like the shareholders, do not internalize that the price of fire sales depends on the amount of fire sales. That is, the executives ignore equation (1).

Using the previous definitions, the executives' problem in (13) leads to the following first order conditions (the subscript mb denotes the executives paid with bonuses):

$$(B_1'(\pi_1^h) + \lambda_{mb}^h)q_{mb}^h + \psi_{mb}^h = \left(B_2'(\pi_2^h)R^{-1} + \mu_{mb}^h\right) b, \tag{A38}$$

$$(\gamma_1 + \lambda_{mb}^l)q_{mb}^l + \psi_{mb}^l = \left(\gamma_2 R^{-1} + \mu_{mb}^l\right) b, \tag{A39}$$

$$\rho_{mb} + p(B_1'(\pi_1^h) + \lambda_{mb}^h)(a^h - R) = (1-p)(\gamma_1 + \lambda_{mb}^l)(R - a^l). \tag{A40}$$

together with the slackness conditions (A7)-(A13).

Following the same arguments as in Proposition 1, we can prove that there are no fire sales in the high state, (i.e. $f_{mb}^h = 0$). This result implies $\lambda_{mb}^h = \mu_{mb}^h = 0$.

Also, by the same reasoning of Proposition 1, given parameter restrictions (8) and (9) we can show $d_{mb} > 0$ and $\psi_{mb}^l = \rho_{mb} = \mu_{mb}^l = 0$. Then we use equation (A40) to obtain:

$$0 < \lambda_{mb}^l = \frac{B_1'(\pi_1^h)p(a^h - R)}{(1-p)(R - a^l)} - \gamma_1.$$

This is the equivalent of equation (A14) in the shareholders'/planner's problem.

From (A39), we obtain another expression for the Lagrange multiplier of fire sales in the low state:

$$\lambda_{mb}^l = \frac{\gamma_2 R^{-1} b}{q_{mb}^l} - \gamma_1.$$

Combining the last two equations we obtain the unique price at which the executives paid with bonuses are willing to participate in fire sales in the low state:

$$q_{mb}^l = \frac{\gamma_2}{B_1'(\pi_1^h)} R^{-1} b \frac{(1-p)(R - a^l)}{p(a^h - R)}. \tag{A41}$$

We see that γ_1 and $B_2(\pi_2)$ play no role in executives' choices.

By comparing (A17) with (A41), we see that:

$$q_{mb}^l = \frac{\gamma_2}{B_1'(\pi_1^h)} q_s^l,$$

and $q_{mb}^l = q_s^l$ only if $\frac{\gamma_2}{B_1'(\pi_1^h)} = 1$. Thus we can obtain our result:

$$\text{if } \frac{\gamma_2}{B_1'(\pi_1^h)} > 1 \text{ then } q_{mb}^l > q_s^l. \tag{A42}$$

Using (A42) and (1) we derive that:

$$\text{if } \frac{\gamma_2}{B_1'(\pi_1^h)} > 1 \text{ then } f_{mb}^l < f_s^l. \tag{A43}$$

In fact, by incorporating the executives' fire sale price (A41) into equation (1), we obtain the executives' optimal fire sales in the bad state:

$$0 < f_{mb}^l = \frac{R^{-1}b}{v}\left(1 - \frac{(1-p)(R-a^l)}{p(a^h-R)}\frac{\gamma_2}{B_1'(\pi_1^h)}\right). \tag{A44}$$

Given $\lambda_{mb}^l > 0$, and equation (A8), the fire sale price determines the ratio of fire sales per unit of debt:

$$\frac{f_{mb}^l}{d_{mb}} = \frac{(R-a^l)}{q_{mb}^l}. \tag{A45}$$

Finally, given (A42), the result $\frac{f_s^l}{d_s} > \frac{f_{mb}^l}{d_{mb}}$ follows.

The ratio (A45) and q_{mb}^l from (A41) give the absolute level of debt:

$$\begin{aligned} d_{mb} &= \frac{f_{mb}^l q_{mb}^l}{(R-a^l)} \\ &= \frac{\gamma_2}{B_1'(\pi_1^h)}\frac{(R^{-1}b)^2}{v(R-a^l)}\frac{(1-p)(R-a^l)}{p(a^h-R)}\left(1 - \frac{(1-p)(R-a^l)}{p(a^h-R)}\frac{\gamma_2}{B_1'(\pi_1^h)}\right). \end{aligned}$$

To show that $\frac{d_{mb}}{d_s} < 1$ for all $\frac{\gamma_2}{B_1'(\pi_1^h)} > 1$, let us now define $x = \frac{\gamma_2}{B_1'(\pi_1^h)}$ in function $h(x)$ from (A37). The same logic and conditions that we applied to show that $\frac{d_{me}}{d_s} < 1$ in the proof of Proposition 2 can be used to show that bail-in bonds decrease the executives' overborrowing relative to the shareholders.

To show that $\frac{\gamma_2}{B_1'(\pi_1^h)} > 1$ reduces the inefficiency, notice that the bank's total profit can be defined as follows:

$$\Pi_{mb}\left(f_{mb}^h, f_{mb}^l, d_{mb}\right) = d_{mb}\left(p(a^h-R) + (1-p)(a^l-R)\frac{R^{-1}b}{q_{mb}^l}\right) + R^{-1}bk_0.$$

And the profit gap relative to the case where the bank is run by the shareholders will be:

$$\Pi_{mb}\left(f_{mb}^h, f_{mb}^l, d_{mb}\right) - \Pi_s\left(f_s^h, f_s^l, d_s\right) = d_{mb}\left(p(a^h-R) + (1-p)(a^l-R)\frac{R^{-1}b}{q_{mb}^l}\right).$$

The gap will be positive as long as $q_{mb}^l > q_s^l$ or, according to our previous result, $\frac{\gamma_2}{B_1'(\pi_1^h)} > 1$.

Comparing q^l_{mb} from (8) and q^l_p from (A22) we obtain the compensation that achieves the social optimal:

$$q^l_{mb} = q^l_p \text{ iff } \frac{\gamma_2}{B_1'(\pi_1^h)} = \frac{1}{2}\left[1 + \frac{p\left(a^h - R\right)}{(1-p)\left(R - a^l\right)}\right] > 1.$$

QED.

Proof of Proposition 4:

Given that $B_2(\pi_2)$ does not enter in (23), we can set $B_2(\pi_2) = 0$. This is a particular feature of bail in bonds: they are state contingent and allow compensation to focus on the incentives to reduce fire sales. In fact, if it were possible to impose penalties, we would make $B_2(\pi_2) < 0$ which implies directly that bail-in compensation is cheaper than the alternatives.

Without loss of generality, we can assume that both compensation structures pay the same amount in the second period if given the low state in the first period:

$$\gamma_2 = \theta_1 + \theta_1^d + \theta_2. \tag{A46}$$

Using the previous relations, and the result $\pi_1^l = 0$ from the previous propositions, the difference between the (deferred) equity and bail-in compensation expenses can be written as follows:

$$W_{me} - W_{mb} = p\pi_1^h\left(\theta_1 + R^{-1}\theta_1^d\right) - pB_1(\pi_1^h) + pR^{-1}\left(\theta_1 + \theta_1^d + \theta_2\right)\pi_2^h. \tag{A47}$$

This shows that the difference between compensation structures comes from the difference in payments in the high state of nature in both periods.

We know that any socially optimal scheme must satisfy:

$$\frac{\gamma_2}{B_1'(\pi_1^h)} = \frac{\theta_1 + \theta_1^d + \theta_2}{\theta_1 + R^{-1}\theta_1^d}. \tag{A48}$$

Given (A48), assumption (A46) implies $B_1'(\pi_1^h) = \theta_1 + R^{-1}\theta_1^d$. Therefore, $B_1(\pi_1^h) = (\theta_1 + R^{-1}\theta_1^d)\pi_1^h$. Replacing $B_1(\pi_1^h)$ in (A47), we obtain:

$$W_{me} - W_{mb} = pR^{-1}\left(\theta_1 + \theta_1^d + \theta_2\right)\pi_2^h > 0.$$

Shareholders will prefer bail-in bonds since they can spare the compensation in the second period when there are no fire sales in the first period. The savings are the probability-weighted present value of the equity payment in the second period in the high state of nature. *QED.*

Proof of Proposition 5

To trace the Pareto Frontier of efficient allocations, we solve the problem of a planner who chooses the efficient allocation of production among the set of feasible allocations and then redistributes the output amongst the agents using taxes or transfers $(T_t^s,\ t=1,2;\ s=l,h)$. The transfers are in zero net supply:

$$T_t^s + \tilde{T}_t^s = 0,\ \forall t, \forall s. \tag{A49}$$

The planner problem traces the Pareto Frontier when maximizing a weighted sum of the expected utility of banks and unskilled investors among the allocations in the feasibility set F. Denoting the social weight of the unskilled investor as $1 \geq \Psi \geq 0$, the social planner solves for

$$U = \underset{d,\{f^s,\tilde{f}^s,T_t^s,\tilde{T}_t^s\}_{s=h,l}}{max} \left\{(1-\Psi)U_B\left(d, f^s, T_t^s\right) + \Psi U_U\left(\tilde{f}^s, \tilde{T}_t^s\right)\right\}, \tag{A50}$$

subject to $\left\{d, f^h, f^l, \tilde{f}^h, \tilde{f}^l\right\} \in F$ and constraints $(A49)$.

In the efficient case, the set of FOC from problem (A50) includes the bank's FOC conditions (A1) to (A3), the price function (1), and the FOCs with respect to the transfers in each state. That is, not internalizing the price function (1) leads to a suboptimal level of fire sales. The excessive costs paid by the unskilled investor are wasted resources. A planner can improve any inefficient allocation by minimizing those wasted resources and redistributing via taxes to make somebody better off.

The Mean-Variance Optimization

The mean-variance shareholder's FOCs (A4)-(A6) become:

$$\begin{aligned}(1-\alpha(1-p)\Delta_s + \lambda_s^h)q_s^h + \psi_s^h &= (R^{-1}(1-\alpha(1-p)\Delta_s) + \mu_s^h)b, \\ (1+\alpha p\Delta_s + \lambda_s^l)q_s^l + \psi_s^l &= (R^{-1}(1+\alpha p\Delta_s) + \mu_s^l)b, \end{aligned} \tag{A51}$$

$$\rho_s + p(1+\lambda_s^h)(a^h - R) - \alpha p(1-p)(a^h - a^l)\Delta_s = (1-p)(1+\lambda_s^l)(R - a^l). \tag{A52}$$

The slackness conditions remain the same as before. Following the same procedure as in the proof of Proposition 1, we obtain that $f_s^h = 0$, and $\lambda_s^h = \mu_s^h = \mu_s^l = \psi_s^h = \psi_s^l = \rho_s = 0$. Equation (29) follows from the FOC (A51). Equation (30) follows from the FOC (A52).

The risk-neutral executive's FOCs (A29)-(A31) become:

$$(\theta_1 + R^{-1}\theta_1^d)(1 - \alpha(1-p)\Delta_{me} + \lambda_{me}^h)q_{me}^h + \psi_{me}^h = \left((\theta_1 + \theta_1^d + \theta_2)(1 - \alpha(1-p)\Delta_{me})R^{-1} + \mu_{me}^h\right) b,$$
$$(\theta_1 + R^{-1}\theta_1^d)(1 + \alpha p\Delta_{me} + \lambda_{me}^l)q_{me}^l + \psi_{me}^l = \left((\theta_1 + \theta_1^d + \theta_2)(1 + \alpha p\Delta_{me})R^{-1} + \mu_{me}^l\right) b,$$
$$\rho_{me} + p(\theta_1 + R^{-1}\theta_1^d + \lambda_{me}^h)(a^h - R) + (1-p)(\theta_1 + R^{-1}\theta_1^d + \lambda_{me}^l)(a^l - R) =$$
$$-\alpha(1-p)p(\theta_1 + R^{-1}\theta_1^d)(a^h - a^l)\Delta_{me}.$$

The slackness conditions remain the same. Following the same procedure as in the proof of Proposition 2, we obtain that $f_{me}^h = 0$, and $\lambda_{me}^h = \mu_{me}^h = \mu_{me}^l = \psi_{me}^l = \rho_{me} = 0$. From the first FOC, it follows that $\psi_{me}^h = (\theta_1^d(1 - R^{-1}) + \theta_2)(1 - \alpha(1-p)\Delta_{me})R^{-1}b$. Equations (32) and (33) follow from the second and third FOCs, respectively.

The risk-neutral executive's FOCs (A38)-(A40) become:

$$(B_1'(\pi_1^h)(1 - \alpha p\Delta_{mb}) + \lambda_{mb}^h)q_{mb}^h + \psi_{mb}^h = \left(B_2'(\pi_2^h)(1 - \alpha p\Delta_{mb})R^{-1} + \mu_{mb}^h\right) b,$$
$$(\gamma_1(1 + \alpha p\Delta_{mb}) + \lambda_{mb}^l)q_{mb}^l + \psi_{mb}^l = \left(\gamma_2(1 + \alpha p\Delta_{mb})R^{-1} + \mu_{mb}^l\right) b,$$
$$\rho_{mb} + p(B_1'(\pi_1^h)(1 - \alpha(1-p)\Delta_{mb}) + \lambda_{mb}^h)(a^h - R) = (1-p)(\gamma_1(1 - \alpha p\Delta_{mb}) + \lambda_{mb}^l)(R - a^l).$$

The slackness conditions remain the same. Following the same procedure as in the proof of Proposition 3, we obtain that $f_{mb}^h = 0$, and $\lambda_{mb}^h = \mu_{mb}^h = \mu_{mb}^l = \psi_{mb}^l = \rho_{mb} = 0$. From the first FOC, it follows that $\psi_{mb}^h = (B_2'(\pi_2) - B_1'(\pi_1))(1 - \alpha(1-p)\Delta_{mb})R^{-1}b$. Equations (35) and (36) follow from the second and third FOCs, respectively.

Proof of Proposition 6

In this case, the First Order Conditions are:

$$(1 + \lambda_s^h)\widehat{q}_s^h + \widehat{\psi}_s^h = (R^{-1} + \widehat{\mu}_s^h)b, \tag{A53}$$
$$(1 + \lambda_s^l)\widehat{q}_s^l + \widehat{\psi}_s^l = (R^{-1} + \widehat{\mu}_s^l)b, \tag{A54}$$
$$\widehat{\rho}_s + p(1 + \widehat{\lambda}_s^h)(a^h - R) = (1-p)(1 + \widehat{\lambda}_s^l)\tau(R - a^l). \tag{A55}$$

We can prove as in Proposition 1 that $\widehat{f}_s^h = 0$, $\widehat{f}_s^l > 0, \widehat{d}_s > 0$, and $\widehat{\lambda}_s^h = \widehat{\mu}_s^h = \widehat{\psi}_s^l = \widehat{\mu}_s^l =$

$\widehat{\rho}_s = 0$. Therefore from $(A55)$, we obtain:

$$\widehat{\lambda}_s^l = \frac{p(a^h - R)}{\tau(1-p)(R - a^l)} - 1. \tag{A56}$$

When there is no guarantee ($\tau = 1$), the shadow price is exactly the same as shareholders' problem in Proposition 1. From $(A54)$, we obtain:

$$\widehat{\lambda}_s^l = \frac{R^{-1}b}{\widehat{q}_s^l} - 1. \tag{A57}$$

Combining $(A56)$ and $(A57)$, we obtain the new fire sale prices when there are government guarantees for shareholders:

$$\widehat{q}_s^l = R^{-1}b\frac{\tau(1-p)(R-a^l)}{p(a^h - R)}. \tag{A58}$$

If $\tau < 1$, the fire sale price is lower with the guarantee because, as we will show, the amount of fire sales is larger. Using (1), we obtain the quantity of fire sales for shareholders:

$$\widehat{f}_s^l = \frac{R^{-1}b}{v}\left[1 - \frac{\tau(1-p)(R-a^l)}{p(a^h-R)}\right]. \tag{A59}$$

Next, we solve for the level of debt for the shareholders. As before, since $\widehat{\lambda}_s^l > 0$, we know that $\widehat{\pi}_1^l = 0$. Then,

$$\frac{\widehat{f}^l}{\widehat{d}} = \frac{\tau\left(R - a^l\right)}{\widehat{q}^l},$$

$$\widehat{d}_s = \frac{\left(R^{-1}b\right)^2}{v}\frac{(1-p)}{p(a^h-R)}\left(1 - \frac{\tau(1-p)(R-a^l)}{p(a^h-R)}\right). \tag{A60}$$

And we can prove that higher government guarantees (lower τ) lead to larger debt and fire sales in the low state of nature. That is, $\frac{\partial \widehat{f}_s^l}{\partial \tau} < 0$, and $\frac{\partial \widehat{d}_s}{\partial \tau} < 0$.

$$\frac{\partial \widehat{f}_s^l}{\partial \tau} = -\frac{R^{-1}b(1-p)(R-a^l)}{vp(a^h-R)} < 0,$$
$$\frac{\partial \widehat{d}_s}{\partial \tau} = -\frac{\left(R^{-1}b\right)^2}{v}\frac{(1-p)}{p(a^h-R)}\frac{(1-p)(R-a^l)}{p(a^h-R)} < 0.$$

For the executive paid with equity, when there are government guarantees, the FOCs are:

$$(\theta_1 + R^{-1}\theta_1^d + \widehat{\lambda}_{me}^h)\widehat{q}_{me}^h + \widehat{\psi}_{me}^h = \left((\theta_1 + \theta_1^d + \theta_2)R^{-1} + \widehat{\mu}_{me}^h\right) b, \tag{A61}$$

$$(\theta_1 + R^{-1}\theta_1^d + \widehat{\lambda}_{me}^l)\widehat{q}_{me}^l + \widehat{\psi}_{me}^l = \left((\theta_1 + \theta_1^d + \theta_2)R^{-1} + \widehat{\mu}_{me}^l\right) b, \tag{A62}$$

$$\widehat{\rho}_{me} + p(\theta_1 + R^{-1}\theta_1^d + \widehat{\lambda}_{me}^h)(a^h - R) = (1-p)(\theta_1 + R^{-1}\theta_1^d + \widehat{\lambda}_{me}^l)\tau(R - a^l). \tag{A63}$$

Following the same steps as before, we obtain that $\widehat{f}_{me}^h = 0$, $\widehat{f}_{me}^l > 0, \widehat{d}_{me} > 0$, with $\widehat{\lambda}_{me}^h = \widehat{\mu}_{me}^h = \widehat{\psi}_{me}^l = \widehat{\mu}_{me}^l = \widehat{\rho}_{me} = 0$.

Then from $(A62)$ and $(A63)$, we obtain:

$$\widehat{\lambda}_{me}^l = (\theta_1 + R^{-1}\theta_1^d)\left(\frac{p(a^h - R)}{\tau(1-p)(R - a^l)} - 1\right). \tag{A64}$$

Now we can solve for the fire sale price:

$$\widehat{q}_{me}^l = R^{-1}b\frac{\tau\left(\theta_1 + \theta_1^d + \theta_2\right)(1-p)(R - a^l)}{(\theta_1 + R^{-1}\theta_1^d)p(a^h - R)}. \tag{A65}$$

The following relationship is the same as before.

$$\frac{\widehat{q}_{me}^l}{\widehat{q}_s^l} = \frac{\theta_1 + \theta_1^d + \theta_2}{\theta_1 + R^{-1}\theta_1^d}. \tag{A66}$$

The above allows us to obtain the level of fire sales using (1):

$$\widehat{f}_{me}^l = \frac{R^{-1}b}{v}\left(1 - \frac{\tau(1-p)(R - a^l)\left(\theta_1 + \theta_1^d + \theta_2\right)}{p(a^h - R)\left(\theta_1 + R^{-1}\theta_1^d\right)}\right). \tag{A67}$$

Now, using $\widehat{\pi}_1^l = 0$, we obtain the level of debt:

$$\widehat{d}_{me} = \frac{\left(\theta_1 + \theta_1^d + \theta_2\right)(R^{-1}b)^2(1-p)}{\left(\theta_1 + R^{-1}\theta_1^d\right)vp(a^h - R)}\left(1 - \frac{\tau(1-p)(R - a^l)\left(\theta_1 + \theta_1^d + \theta_2\right)}{p(a^h - R)\left(\theta_1 + R^{-1}\theta_1^d\right)}\right) \tag{A68}$$

Optimality implies:

$$\frac{\widehat{d}_{me}}{d_p} = 1. \tag{A69}$$

Denoting $x = \frac{\theta_1+\theta_1^d+\theta_2}{\theta_1+R^{-1}\theta_1^d}$, and $b = \frac{(1-p)(R-a^l)}{p(a^h-R)}$, then $(A69)$ becomes:

$$4bx\frac{(1-\tau bx)}{1-b^2} = 1. \tag{A70}$$

This equation has the following solution:

$$x = \frac{1 \pm \sqrt{1-\tau\left(1-b^2\right)}}{2\tau b}.$$

The non-negativity in the square root is satisfied because $\tau \leq 1$, and because parameter restriction (8) ensures $\frac{1}{1-b^2} > 1$. We select the only root larger than one because $x = \frac{\theta_1+\theta_1^d+\theta_2}{\theta_1+R^{-1}\theta_1^d} \geq 1$. That root is:

$$x = \frac{1 + \sqrt{1-\tau\left(1-b^2\right)}}{2\tau b}.$$

It imposes the restriction $0 < \tau < \frac{b^2+4b-1}{4b^2}$.

Taking the derivative of the optimal ratio of compensation, over τ, we obtain:

$$\frac{\partial x}{\partial \tau} = \frac{\frac{1}{2}\left[1-\tau\left(1-b^2\right)\right]^{-\frac{1}{2}}\left(b^2-1\right)2\tau b - \left(1+\sqrt{1-\tau\left(1-b^2\right)}\right)2b}{\left(2\tau b\right)^2} < 0. \tag{A71}$$

That is, when government guarantees are higher (lower τ) there should be lower rewards from debt. To show the previous result we used (8) that implies that $b^2 - 1 < 0$. Then, since $0 \leq \tau \leq 1$, we know that $1-\tau\left(1-b^2\right) > 0$. Therefore the optimal ratio decreases with τ.

With the government guarantee, the FOCs for compensation structure #2 are:

$$(B_1'(\widehat{\pi}_1^h) + \widehat{\lambda}_{mb}^h)\widehat{q}_{mb}^h + \widehat{\psi}_{mb}^h = \left(B_2'(\widehat{\pi}_2^h)R^{-1} + \widehat{\mu}_{mb}^h\right) b, \tag{A72}$$

$$(\gamma_1 + \widehat{\lambda}_{mb}^l)\widehat{q}_{mb}^l + \widehat{\psi}_{mb}^l = \left(\gamma_2 R^{-1} + \widehat{\mu}_{mb}^l\right) b, \tag{A73}$$

$$\widehat{\rho}_{mb} + p(B_1'(\widehat{\pi}_1^h) + \widehat{\lambda}_{mb}^h)(a^h - R) = (1-p)(\gamma_1 + \widehat{\lambda}_{mb}^l)\tau(R - a^l). \tag{A74}$$

Again, we obtain that $\widehat{f}_{mb}^h = 0$, $\widehat{f}_{mb}^l > 0, \widehat{d}_{mb} > 0$, with $\widehat{\lambda}_{mb}^h = \widehat{\mu}_{mb}^h = \widehat{\psi}_{mb}^l = \widehat{\mu}_{mb}^l = \widehat{\rho}_{mb} = 0$.

Then, from $(A73)$ and $(A74)$, we obtain:

$$\widehat{\lambda}_{mb}^l = \frac{\gamma_2 R^{-1} b}{\widehat{q}_{mb}^l} - \gamma_1 = \frac{B_1'(\pi_1^h)p(a^h - R)}{\tau(1-p)(R-a^l)} - \gamma_1. \tag{A75}$$

The fire sale price is:

$$\widehat{q}_{mb}^l = \frac{\gamma_2}{B_1'(\pi_1^h)} R^{-1} b \frac{\tau(1-p)(R-a^l)}{p(a^h - R)}, \tag{A76}$$

which is lower than when there are no guarantees. Using (1), we obtain the following level of fire sales:

$$\widehat{f}^l_{mb} = \frac{R^{-1}b}{v}\left(1 - \frac{\tau(1-p)(R-a^l)}{p(a^h-R)}\frac{\gamma_2}{B_1'(\pi_1^h)}\right). \tag{A77}$$

The following link still holds:

$$\widehat{q}^l_{mb} = \frac{\gamma_2}{B_1'(\pi_1^h)}\widehat{q}^l_s.$$

Then, if $\frac{\gamma_2}{B_1'(\pi_1^h)} > 1$, we obtain $\widehat{q}^l_{mb} > \widehat{q}^l_s$, and $\widehat{f}^l_{mb} > \widehat{f}^l_s$. The level of debt is:

$$\widehat{d}_{mb} = \frac{\gamma_2}{B_1'(\pi_1^h)}\frac{(R^{-1}b)^2}{v}\frac{(1-p)}{p(a^h-R)}\left(1 - \frac{\tau(1-p)(R-a^l)}{p(a^h-R)}\frac{\gamma_2}{B_1'(\pi_1^h)}\right), \tag{A78}$$

and the ratio of debt under compensation structure #2 relative to the planner is:

$$\frac{\widehat{d}_{mb}}{d_p} = \frac{\gamma_2}{B_1'(\pi_1^h)}\frac{4(1-p)(R-a^l)}{p(a^h-R)}\frac{\left(1 - \frac{\tau(1-p)(R-a^l)}{p(a^h-R)}\frac{\gamma_2}{B_1'(\pi_1^h)}\right)}{\left(1 - \left(\frac{(1-p)(R-a^l)}{p(a^h-R)}\right)^2\right)}. \tag{A79}$$

The optimal compensation is when $\frac{\widehat{d}_{mb}}{d_p} = 1$. We denote $y = \frac{\gamma_2}{B_1'(\pi_1^h)}$, and $z = \frac{(1-p)(R-a^l)}{p(a^h-R)}$. Then, using the same methodology as we used before to analyze compensation structure #1, we obtain

$$y = \frac{1 + \sqrt{1 - \tau\left(1 - z^2\right)}}{2\tau z}. \tag{A80}$$

With $\frac{\partial y(\tau)}{\partial \tau} < 0$, the optimal compensation ratio decreases with τ.